Replication of Frank Smets and Raf Wouters (2003) 'An estimated dynamic stochastic general equilibrium model of the Euro area', Journal of the European Economic Association

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Dynamic equations:

$$c_t = \frac{h}{1+h}\,c_{t-1} + c_{t+1}\left(1 - \frac{h}{1+h}\right) - \frac{1-h}{(1+h)\,\sigma}\,(R_t - \pi_{t+1}) + \frac{1-h}{(1+h)\,\sigma}\,\epsilon^b_t \tag{1}$$

$$i_t = \epsilon^I{}_t + \frac{1}{1+\beta}\,i_{t-1} + i_{t+1}\left(1 - \frac{1}{1+\beta}\right) + \frac{1}{(1+\beta)\,\varphi}\,Q_t \tag{2}$$

$$Q_t = \eta^Q{}_t + \frac{\left(\frac{1}{\beta} - 1 + \delta\right) r^k{}_{t+1}}{\frac{1}{\beta} - 1 + \delta + 1 - \delta} + Q_{t+1}\,\frac{1-\delta}{1 + \frac{1}{\beta} - 1 + \delta - \delta} - (R_t - \pi_{t+1}) \tag{3}$$

$$k_t = (1-\delta)\,k_{t-1} + \delta\,i_{t-1} \tag{4}$$

$$\pi_t = \frac{\beta\,\pi_{t+1} + \iota_p\,\pi_{t-1} + \frac{(1-\beta\,\xi_p)\,(1-\xi_p)}{\xi_p}\,MC_t}{1 + \beta\,\iota_p} + \eta^\pi{}_t \tag{5}$$

$$MC_t = \alpha\,r^k{}_t + (1-\alpha)\,w_t - a_t \tag{6}$$

$$w_t = \eta^w{}_t + w_{t+1}\,\beta\,\frac{1}{1+\beta} + \frac{1}{1+\beta}\,w_{t-1} + \pi_{t+1}\,\beta\,\frac{1}{1+\beta} - \frac{1 + \beta\,\iota_w}{1+\beta}\,\pi_t + \frac{\iota_w}{1+\beta}\,\pi_{t-1} - \frac{(1-\xi_w)\,(1-\beta\,\xi_w)}{(1+\beta)\,\xi_w\left(1 + \frac{(1+\lambda_w)\,\sigma_l}{\lambda_w}\right)}\left(w_t - \sigma_l\,L_t - \frac{\sigma\,(c_t - h\,c_{t-1})}{1-h} + \epsilon^L{}_t\right) \tag{7}$$

$$L_t = k_{t-1} + r^k{}_t\,\frac{1 + \psi^{-1}}{\psi^{-1}} - w_t \tag{8}$$

$$y_t = k_c\,c_t + k_i\,i_t + \epsilon^G{}_t \tag{9}$$

$$\psi^{-1}\,z_t = r^k{}_t \tag{10}$$

$$y_t = \Phi_p\left(a_t + k_t\,\alpha + r^k{}_t\,\alpha\,\frac{1}{\psi^{-1}} + (1-\alpha)\,L_t\right) \tag{11}$$

$$R_t = \rho_R\,R_{t-1} + (1-\rho_R)\left(\bar\pi_t + r_\pi\,(\pi_{t-1} - \bar\pi_t) + r_Y\,(y_t - y^p{}_t)\right) + r_{\Delta\pi}\,(\pi_t - \pi_{t-1}) + r_{\Delta y}\left(y_t - y^p{}_t - (y_{t-1} - y^p{}_{t-1})\right) + \eta^r{}_t \tag{12}$$

$$E_t - E_{t-1} = E_{t+1} - E_t + (L_t - E_t) \, \frac{(1 - \xi_e) \, (1 - \beta \, \xi_e)}{\xi_e} \tag{13}$$

$$a_t = \rho_a \, a_{t-1} + \eta^a{}_t \tag{14}$$

$$\epsilon^L{}_t = \rho_L \, \epsilon^L{}_{t-1} + \eta^L{}_t \tag{15}$$

$$\epsilon^b{}_t = \rho_b \, \epsilon^b{}_{t-1} + \eta^b{}_t \tag{16}$$

$$\epsilon^I{}_t = \rho_I \, \epsilon^I{}_{t-1} + \eta^i{}_t \tag{17}$$

$$\bar{\pi}_t = \rho_\pi \, \bar{\pi}_{t-1} + \eta^\pi{}_t \tag{18}$$

$$\epsilon^G{}_t = \rho_g \, \epsilon^G{}_{t-1} + \eta^G{}_t \tag{19}$$

$$c^p{}_t = \frac{1 - h}{(1 + h) \, \sigma} \, \epsilon^b{}_t + \frac{h}{1 + h} \, c^p{}_{t-1} + c^p{}_{t+1} \left(1 - \frac{h}{1 + h} \right) - \frac{1 - h}{(1 + h) \, \sigma} \, R^p{}_t \tag{20}$$

$$i^p{}_t = \epsilon^I{}_t + \frac{1}{1 + \beta} \, i^p{}_{t-1} + i^p{}_{t+1} \left(1 - \frac{1}{1 + \beta} \right) + \frac{1}{(1 + \beta) \, \varphi} \, Q^p{}_t \tag{21}$$

$$Q^p{}_t = \frac{\left(\frac{1}{\beta} - 1 + \delta \right) \, r^{k,p}{}_{t+1}}{\frac{1}{\beta} - 1 + \delta + 1 - \delta} + Q^p{}_{t+1} \, \frac{1 - \delta}{1 + \frac{1}{\beta} - 1 + \delta - \delta} - R^p{}_t \tag{22}$$

$$k^p{}_t = (1 - \delta) \, k^p{}_{t-1} + \delta \, i^p{}_{t-1} \tag{23}$$

$$y^p{}_t = \Phi_p \left(a_t + \alpha \, k^p{}_t + \alpha \, \frac{1}{\psi^{-1}} \, r^{k,p}{}_t + (1 - \alpha) \, L^p{}_t \right) \tag{24}$$

$$y^p{}_t = \epsilon^G{}_t + k_c \, c^p{}_t + k_i \, i^p{}_t \tag{25}$$

$$w^p{}_t = \sigma_l \, L^p{}_t + \frac{\sigma \, (c^p{}_t - h \, c^p{}_{t-1})}{1 - h} - \epsilon^L{}_t \tag{26}$$

$$L^p{}_t = k^p{}_{t-1} + \frac{1 + \psi^{-1}}{\psi^{-1}} \, r^{k,p}{}_t - w^p{}_t \tag{27}$$

$$E^p{}_t - E^p{}_{t-1} = E^p{}_{t+1} - E^p{}_t + \frac{(1 - \xi_e) \, (1 - \beta \, \xi_e)}{\xi_e} \, (L^p{}_t - E^p{}_t) \tag{28}$$

$$0 = \alpha \, r^{k,p}{}_t + (1 - \alpha) \, w^p{}_t - a_t \tag{29}$$

Definitions of variables and parameters

Table 1: Endogenous

Variable	LaTeX	Description
c	c	Consumption
g	ϵ^G	Exogenous spending
inve	i	Investment
kp	k	Installed capital
lab	L	Labour supply
mc	MC	Real marginal cost
pinf	π	Inflation
pk	Q	Tobin Q
r	R	Nominal interest rate
rk	r^k	Rental cost of capital
w	w	Real wage
y	y	Output
a	a	Productivity shock
b	ϵ^b	Discount factor shock
sinve	ϵ^I	Investment-specific shock
ls	ϵ^L	Labour supply shock
zcap	z	Capital utilisation
pinfbar	$\bar{\pi}$	Inflation target
empl	E	Employment in persons
cf	c^p	Consumption, flexible ecomomy
invef	i^p	Investment, flexible ecomomy
pkf	Q^p	Tobin Q, flexible ecomomy
kpf	k^p	Installed capital, flexible ecomomy
wf	w^p	Real wage, flexible ecomomy
yf	y^p	Output, flexible ecomomy
rf	R^p	Nominal interest rate, flexible ecomomy
rkf	$r^{k,p}$	Rental cost of capital, flexible ecomomy
labf	L^p	Labour supply, flexible ecomomy
emplf	E^p	Employment in persons, flexible ecomomy

Table 2: Exogenous

Variable	LaTeX	Description
ea	η^a	Productivity shock innovation
eb	η^b	Discount factor shock innovation
eg	η^G	Exogenous spending shock innovation
einve	η^i	Investment-specific shock innovation
eqs	η^Q	Equity premium shock
em	η^r	Monetary policy shock
epinf	η^π	Price markup shock
ew	η^w	Wage markup shock
els	η^L	Labour supply shock innovation
epb	η^π	Inflation target shock

Table 3: Parameters

Variable	LaTeX	Description
calfa	α	Output elasticity w.r.t. capital
cbeta	β	Discount factor
czcap	ψ^{-1}	Elasticity of the capital utilization cost function
csadjcost	φ	Capital adjustment cost
ctou	δ	Depreciation rate
chabb	h	Habits in consumption
cfc	Φ_p	Fixed cost
cprobp	ξ_p	Calvo price rigidity
cindp	ι_p	Indexation on prices
csigma	σ	Risk aversion
csigl	σ_l	Labour supply utility
cindw	ι_w	Indexation on wages
cprobw	ξ_w	Calvo wage rigidity
crr	ρ_R	Interest rate smoothing
crpi	r_π	Taylor rule inflation coefficient
crdpi	$r_{\Delta\pi}$	Taylor rule d(inflation) coefficient
cry	r_Y	Taylor rule output gap coefficient
crdy	$r_{\Delta y}$	Taylor rule d(output gap) coefficient
clandaw	λ_w	Wage markup -1
crhoa	ρ_a	Technology shock persistence
crhob	ρ_b	Discount factor shock persistence
crhog	ρ_g	Exogenous spending shock persistence
crhols	ρ_L	Labour supply shock persistence
crhoinve	ρ_I	Investment-specific shock persistence
crhopb	ρ_π	Inflation target shock persistence
ccy	k_c	Steady-state consumption share in output
ciy	k_i	Steady-state investment share in output
cprobe	ξ_e	Calvo employment rigidity