Simulator of invertible and non-invertible MA(q) models and Eviews add-in

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Abstract

The identification of MA(q) processes using second order techniques (such as OLS, ML and Box-Jenkins) estimates the invertible representation and ignores the non-invertible representations of a time series. This add-in seeks to simulate and find the data generating mechanism of all representations of a MA(q). In general there are 2^q representations of a MA(q) and only one of them is invertible. In this document it is explained how to use the simulator of invertible and non-invertible MA(q) models using an add-in. This document contains a theoretical and empirical explanation; several examples of simulations are performed. The importance of this subject is discussed in Granger et. Al (1978), Hamilton (1994), Hallin (1986), Huang et. Al. (2000), Ramsey (1991), Ramsey & Montenegro (1988) and Plosser et. Al (1977).

Keywords: Moving average models, time series and Box-Jenkins.

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1. Second order moments of invertible and non-invertible MA(q) process

An MA(q) process it is said to be invertible if it can be represented as a AR(∞) process. Let x_t be an MA(q) process defined by:

$$x_t = e_t + \beta_1 e_{t-1} + \cdots + \beta_q e_{t-q} = e_t(1 + \beta_1 L + \cdots + \beta_q L^q)$$

Where L represents the lag operator and e_t is a white noise process with zero mean. By the fundamental theorem of algebra, any polynomial of degree q possesses q roots (real or complex). Therefore:

$$x_t = e_t(1 - \lambda_1 L)(1 - \lambda_2 L) \dots (1 - \lambda_q L)$$

Where λ_i represents the inverse of the root of the MA(q) lag polynomial. Therefore if $|\lambda_i| < 1$ then the MA(q) it is invertible, the AR representation is given by:

$$\frac{x_t}{(1 - \lambda_1 L)(1 - \lambda_2 L) \dots (1 - \lambda_q L)} = x_t(1 + \lambda_1 L + (\lambda_1 L)^2 + \cdots) \dots (1 + \lambda_q L + \left(\lambda_q L\right)^2 + \cdots)$$

There are 2^q representations of an MA(q) process with the same autocorrelation function, of this 2^q representations only one is invertible and the rest are non-invertible. The identification of MA(q) models is seriously affected by this if we use second order techniques such as OLS, ML and Box-Jenkins to identify the coefficients of the MA model; because we cannot differentiate between the invertible and non-invertible representations. The OLS and Maximum Likelihood estimators always estimate the invertible representation; even if the true process is non-invertible see Hamilton (1994).

The add-in calculates MA(q) representations by **i) estimating the inverse roots using eviews if the MA parameters are provided or ii) plugging in the inverse roots directly**. Note that the second option is more precise because it does not involve any random variable estimation. By using the inverse roots λ_i the add-in calculates the MA(q) representations using the following expressions.

$$x_t = e_t(1 - \lambda_1 L)(1 - \lambda_2 L) \dots \left(1 - \lambda_q L\right) = e_t + \beta_1 e_{t-1} + \cdots + \beta_q e_{t-q}$$

$$\beta_1 = -(\lambda_1 + \lambda_2 + \cdots + \lambda_q)$$

$$\beta_2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \cdots + \lambda_1\lambda_q + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \cdots + \lambda_2\lambda_q + \cdots$$

$$\beta_3 = \lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \cdots + \lambda_1\lambda_2\lambda_q + \lambda_1\lambda_3\lambda_4 + \cdots + \lambda_1\lambda_3\lambda_q + \cdots + \lambda_2\lambda_3\lambda_4 + \cdots$$

$$\vdots$$

$$\beta_{12} = \lambda_1\lambda_2\lambda_3\lambda_4\lambda_5\lambda_6\lambda_7\lambda_8\lambda_9\lambda_{10}\lambda_{11}\lambda_{12} + \cdots + \cdots + \cdots + \cdots$$

That is each β_j coefficient of the MA(q) model is calculated as the permutation of order $j-th$. **The add-in only supports the calculation up to order twelve**. **The add-in only admits real roots, therefore the inverse roots λ_i must be real to use the add-in[2].** For example, consider de following MA(1) model:

$$x_t = e_t + \beta_1 e_{t-1}$$

With $|\beta_1| < 1$, therefore the MA(1) process is invertible. The autocorrelation function of the process is given by the following expression; consider that x_t has zero mean.

$$r(1) = \frac{R(1)}{R(0)} = \frac{E(x_t x_{t-1})}{E(x_t^2)} = \frac{\beta_1\sigma_e^2}{\sigma_e^2 + \sigma_e^2\beta_1^2} = \frac{\beta_1}{1+\beta_1^2}$$

$$r(\tau) = 0 \quad for\, \tau > 1$$

Now, consider the non-invertible MA(1):

$$x_t = e_t + \frac{1}{\beta_1} e_{t-1}$$

For this process, the autocorrelation is given by:

[2] In case that the user plug in the coefficients of an MA(q) model with real roots exists the possibility that the estimation of roots be complex because of statistical error, in that case it is suggested to plug in the inverse roots directly instead of using the coefficients.

$$r(1) = \frac{R(1)}{R(0)} = \frac{E(x_t x_{t-1})}{E(x_t^2)} = \frac{\frac{1}{\beta_1}\sigma_e^2}{\sigma_e^2 + \frac{\sigma_e^2}{\beta_1^2}} = \frac{\frac{1}{\beta_1}}{\frac{1+\beta_1^2}{\beta_1^2}} = \frac{\beta_1}{1+\beta_1^2}$$

$$r(\tau) = 0 \quad for\, \tau > 1$$

Therefore, by using Box-Jenkins one cannot differentiate between the invertible and non-invertible representations.

2. Eviews Add-in

In this section some examples using the add-in are given. The add-in can be used using two interfaces i) point and clicking and ii) command. With any of the interfaces the add-in calculates all the MA(q) representations and a simulation of the data for each MA representation.

The first interface is showed is figure 1, the user has to write the coefficients of the MA(q) model separated by spaces e.g. -0.833 0.1666 to write the model $x_t = e_t - 0.833e_{t-1} + 0.166e_{t-2}$ in the first blank box to estimate the inverse roots of the MA, the estimation is performed using a simulation and 100000 observations. Instead of estimating the inverse roots the user can plug in the roots directly. In addition, the user of the add-in can select to estimate the correlograms and the equations (by M.L) of all the MA representations.

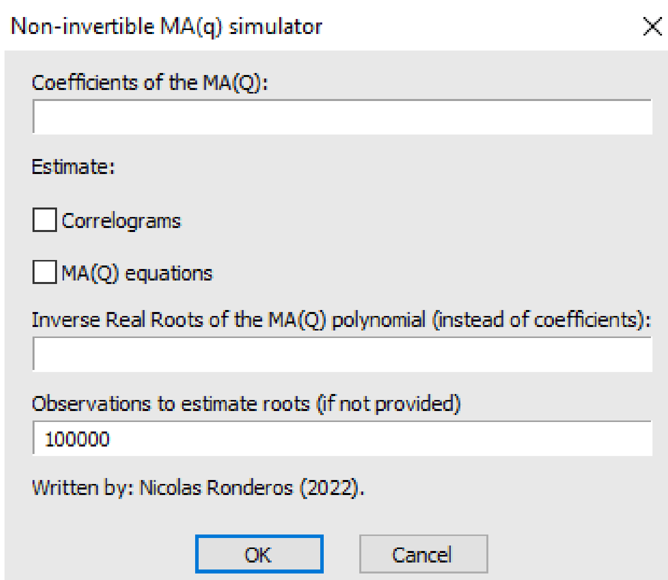

Figure 1 – Point and click interface

The global commands of the add-in can be found in table 1.

noninv_ma(options)

Option	Description
Coefs=# # #	Insert the MA coefficients to estimate the inverse roots of the MA polynomial
Roots=# # #	Insert the inverse roots directly
Correl	Estimate the correlogram for each MA representation. By default, the add-in dos not estimate the correlograms.
equation	Estimate the MA equation for each MA representation. By default, the add-in dos not estimate the equations.
n	Number of observations to estimate the inverse roots of the MA polynomial. By default the add-in use 100.000 observations.

Table 1 – Commands

Example 1 – MA(1)

To estimate all the representations of the following MA(1) model one can write *noninv_ma(coefs=0.5)* to estimate the inverse roots or use *noninv_ma(roots=0.5)* to use the inverse roots directly.

$$x_t = e_t + 0.5e_{t-1}$$

The results for each command are showed in table 2.

noninv_ma(coefs=0.5)

Representation	MA(Q) equation	Inverse roots
1.000000	X1t=et+0.51e(-1)	(-0.51)
2.000000	X2t=et+1.97e(-1)	(-1.97)

noninv_ma(roots=0.5)

Representation	MA(Q) equation	Inverse roots
1.000000	X1t=et+-0.5e(-1)	(0.5)
2.000000	X2t=et+-2e(-1)	(2)

Table 2 – Representations of MA(1) model

The command output also shows three time series x_{1t}, x_{2t} and $e_t \sim nid(0,1)$ correspond to each MA representation and the normal distributed residuals to simulate each representation. The time series are showed in figure 2, clearly the two representations greatly differ.

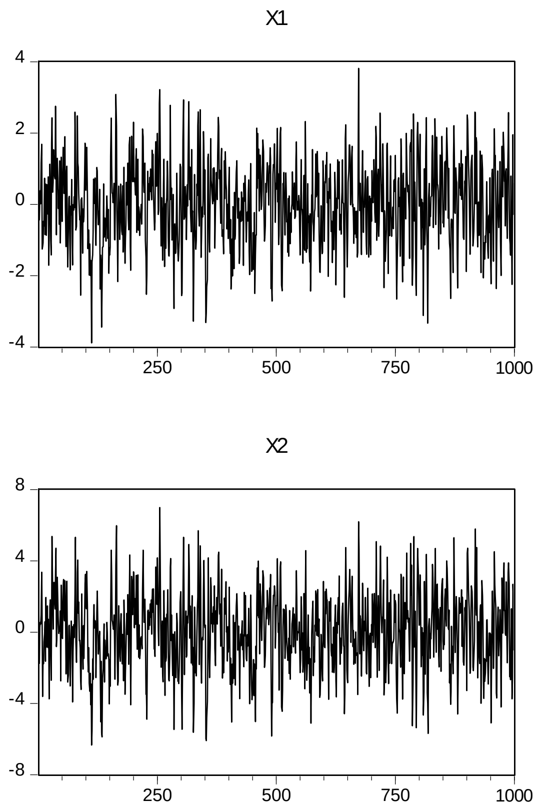

Figure 2 – time series representations and residuals

To estimate the correlogram and the equation for each representation you can use the command *noninv_ma(roots=0.5,correl,equations).* The correlograms of the two representations are equal and the estimation of the non-invertible representation x_{2t} is identified as the invertible representation, even if the true representation is non-invertible.

Example 2 – MA(2)

Consider the following invertible MA(2) model with inverse roots $(0.5, 0.33)$.

$$x_t = e_t - 0.833e_{t-1} + 0.1666e_{t-2}$$

With the commands *noninv_ma(coefs=-0.8333 0.1666)* and *noninv_ma(roots=0.5 0.33)* the add-in calculates the following representations in table 3.

noninv_ma(coefs=-0.8333 0.1666)

Representation	MA(Q) equation	Inverse roots
1.000000	X1t=et+-0.83e(-1)+0.16e(-2)	(0.49,0.33)
2.000000	X2t=et+-3.51e(-1)+1.49e(-2)	(0.49,3.01)
3.000000	X3t=et+-2.35e(-1)+0.67e(-2)	(2.02,0.33)
4.000000	X4t=et+-5.03e(-1)+6.08e(-2)	(2.02,3.01)

noninv_ma(roots=0.5 0.33)

Representation	MA(Q) equation	Inverse roots
1.000000	X1t=et+-0.83e(-1)+0.17e(-2)	(0.5,0.33)
2.000000	X2t=et+-3.53e(-1)+1.52e(-2)	(0.5,3.03)
3.000000	X3t=et+-2.33e(-1)+0.66e(-2)	(2,0.33)
4.000000	X4t=et+-5.03e(-1)+6.06e(-2)	(2,3.03)

Table 3 – Representations of MA(2) model

The previous model is used in Ramsey & Montenegro (1988) as an example of invertible and non-invertible MA models.

Example 3 – MA(3)

Consider the non-invertible model MA(3)

$$x_t = e_t - 7e_{t-1} - e_{t-2} + 2e_{t-3}$$

Clearly as the order of the MA(q) model grows there are more equivalent representations and to identify them became more complex. The representations are showed in table 4.

noninv_ma(coefs=-7 -1 2,n=1000000)

Representation	MA(Q) equation	Inverse roots
1.000000	X1t=et+-0.04e(-1)+-0.3e(-2)+0.04e(-3)	(-0.58,0.48,0.14)
2.000000	X2t=et+-6.91e(-1)+-1e(-2)+1.97e(-3)	(-0.58,0.48,7.01)
3.000000	X3t=et+-1.63e(-1)+-1e(-2)+0.17e(-3)	(-0.58,2.07,0.14)
4.000000	X4t=et+-8.5e(-1)+9.24e(-2)+8.5e(-3)	(-0.58,2.07,7.01)
5.000000	X5t=et+1.09e(-1)+-1e(-2)+0.12e(-3)	(-1.71,0.48,0.14)
6.000000	X6t=et+-5.78e(-1)+-9.46e(-2)+5.79e(-3)	(-1.71,0.48,7.01)
7.000000	X7t=et+-0.5e(-1)+-3.5e(-2)+0.51e(-3)	(-1.71,2.07,0.14)
8.000000	X8t=et+-7.38e(-1)+-1.01e(-2)+24.92e(-3)	(-1.71,2.07,7.01)

Table 4 – Representations of MA(3) model

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