adequately describes the original monthly hotel room averages in Table 11.1. A necessary (but not sufficient) stationarity condition on the parameters ϕ_1, ϕ_3, and ϕ_5 in the nonseasonal autoregressive operator

$$\phi_5(B) = (1 - \phi_1 B - \phi_3 B^3 - \phi_5 B^5)$$

is

$$\phi_1 + \phi_3 + \phi_5 < 1$$

Moreover, the invertibility condition on the parameter $\theta_{1,12}$ in the seasonal moving average operator of order 1

$$\theta_1(B^{12}) = (1 - \theta_{1,12} B^{12})$$

is (from Table 12.3)

$$|\theta_{1,12}| < 1$$

Note that the preliminary point estimates $\hat{\phi}_1 = .1, \hat{\phi}_3 = .1, \hat{\phi}_5 = .1$, and $\hat{\theta}_{1,12} = .1$ satisfy these stationarity and invertibility conditions, and note that

$$\hat{\phi}_1 = .23242 \qquad \hat{\phi}_3 = -.22301 \quad \hat{\phi}_5 = -.15263 \quad \text{and} \quad \hat{\theta}_{1,12} = .47634$$

(which are the final least squares point estimates of ϕ_1, ϕ_3, ϕ_5 and $\theta_{1,12}$ in Figure 11.9) also satisfy the stationarity and invertibility conditions.

12.2 INTERVENTION MODELS

Intervention models are used when exceptional external events, called interventions, affect the variable to be forecasted. Examples of interventions are strikes, natural disasters, and policy changes. As demonstrated in the following examples, we use special types of dummy variables called step functions and impulse functions to build intervention models.

EXAMPLE 12.5 In March 1974 the Cincinnati Bell Telephone Company initiated a policy intended to reduce the frequency of local directory assistance calls. According to this policy, each subscriber is allowed three such calls each month and then is charged 20 cents for each additional such call. Prior to March 1974 there had been no such charge. In Table 12.4 we present the monthly average number of directory assistance calls per day (Sundays excluded) made by Cincinnati Bell subscribers from January 1962 to December 1976. There are 180 observations in this table, and the number of calls in March 1974 is observation 147. In examining the plot of these data in Figure 12.1, we see that the effect of the new charge was to reduce substantially the number of calls.

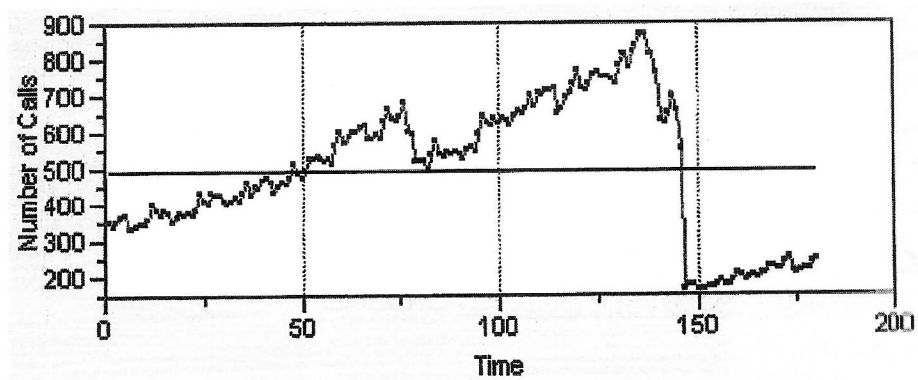
To estimate the size of this reduction and to develop a model for forecasting future

TABLE 12.4 Cincinnati Directory Assistance, Monthly Average Calls per Day (In Units of 100 Calls), From January 1962 to December 1976 (Read Left to Right)

350	339	351	364	369	331	331	340	346	341	357	398
381	367	383	375	353	361	375	371	373	366	382	429
406	403	429	425	427	409	402	409	419	404	429	463
428	449	444	467	474	463	432	453	462	456	474	514
489	475	492	525	527	533	527	522	526	513	564	599
572	587	599	601	611	620	579	582	592	581	630	663
638	631	645	682	601	595	521	521	516	496	538	575
537	534	542	538	547	540	526	548	555	545	594	643
625	616	640	625	637	634	621	641	654	649	662	699
672	704	700	711	715	718	652	664	695	704	733	772
716	712	732	755	761	748	748	750	744	731	782	810
777	816	840	868	872	811	810	762	634	626	649	697
657	549	162	177	175	162	161	165	170	172	178	186
178	178	189	205	202	185	193	200	196	204	206	227
225	217	219	236	253	213	205	210	216	218	235	241

Source: Dr. A. J. McSweeny, Department of Psychology, University of West Virginia. See McSweeny (1978).

FIGURE 12.1
JMP IN plot of the directory assistance call data in Table 12.4



Step 1: Find a Box–Jenkins model describing the time series values observed before (or after) the intervention.

Consider the 146 observations before the intervention. It can be verified that taking the seasonal difference of the regular differences of the y_t values produces stationary z_t values and that the stationary values should be described by a seasonal moving average model. A model describing the observations before the intervention is

$$z_t = (1 - \theta_{1,12}B^{12})a_t \quad \text{where} \quad z_t = (1 - B)(1 - B^{12})y_t$$
$$= y_t - y_{t-1} - y_{t-12} + y_{t-13}$$

Step 2: Using appropriately defined dummy variables, find a regression model describing the intervention.

Figure 12.1 indicates that the charge (in March 1974) abruptly and permanently decreased the number of monthly average calls per day by a particular amount. The rate of increase in calls after March 1974 [illegible]

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previously used such a procedure in Section 11.3 to find a Box–Jenkins model describing the error terms of a time series regression model.

More General Intervention Models

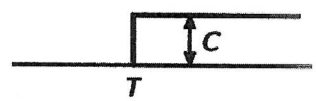
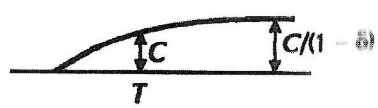
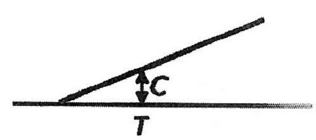
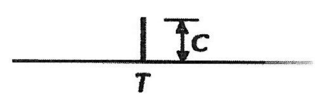
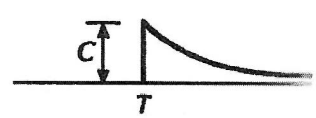
The functions CS_t, C_1P_{1t}, and C_2P_{2t} used in the previous examples are called **response functions** for modeling interventions. Figure 12.4 summarizes six useful response functions and the types of intervention responses that they imply. To understand these functions, suppose that an intervention occurs at time T. The first three response functions employ the step function

$$S_t = \begin{cases} 0 & \text{if } t < T \text{ (before the intervention)} \\ 1 & \text{if } t \geq T \text{ (at and after the intervention)} \end{cases}$$

The first function in Figure 12.4, CS_t, was used in Example 12.5 to model the abrupt and permanent decrease in the number of monthly average calls per day associated with the new directory assistance charge. If we thought that the decrease in the number of monthly average calls was gradual and led to a permanent change (excluding such factors as population growth), we would utilize the second function, $[C/(1-\delta B)]S_t$. Therefore, the model would be

$$z_t = \frac{C}{(1-\delta B)} z_t(S) + \varepsilon_t$$

FIGURE 12.4
Forms of intervention responses

Type of Response	Response Function	Typical Diagram
Abrupt start and permanent effect	CS_t	
Gradual start and permanent effect	$\frac{C}{1-\delta B}S_t$	
Linearly changing without limit	$\frac{C}{1-B}S_t$	
Abrupt start and abrupt decay	CP_t	
Abrupt start and gradual decay	$\frac{C}{1-\delta B}P_t$	
Abrupt start and gradual decay to a permanent level	$\frac{C_1}{1-\delta B}P_t + \frac{C_2}{1-B}P_t$