

$$\log[Std(\beta_{imt}^b)] = \mu_m + H_{mt} + v_{mt}$$

$$H_{mt} = \Phi_m H_{mt-1} + \eta_{mt}$$

Using the assumption on $Std_c(\beta_{mt})$, it can be written:

$$\log [Std_c(\beta_{mt})] = \mu_m + v_{mt} \quad (5)$$

where $\mu_m = E(\log [Std_c(\beta_{mt})])$ and $v_{mt} \sim iid(0, \sigma_{mv}^2)$, and then:

$$\log [Std_c(\beta_{mt}^b)] = \mu_m + H_{mt} + v_{mt} \quad (6)$$

where $H_{mt} = \log(1 - h_{mt})$.

Moving forward, H_{mt} is assumed to follow a mean zero AR(1) process:

$$H_{mt} = \phi_m H_{mt-1} + \eta_{mt} \quad (7)$$

where $\eta_{mt} \sim iid(0, \sigma_{m\eta}^2)$.

This is now a standard state space model with Eq.6 as the measurement equation and Eq.7 as the transition equation. It can be estimated with the Kalman filter.