Conditional VAR forecasting

(technical appendix of the confcast add-in)

Let's consider the following structural VAR with N variables:

$$Y_t = BY_{t-1} + A_0\varepsilon_t \tag{1}$$

If we iterate the equation m times forward, we get the following equation:

$$Y_{t+m} = B^{m+1}Y_{t-1} + A_0\sum_{i=0}^{m} B^i\,\varepsilon_{t+m-i} \tag{2}$$

From this equation we observe that forecast Y_{t+m} can be decomposed into two components: unconditional forecast, component with structural shocks. We can rearrange the equation:

$$Y_{t+m} - B^{m+1}Y_{t-1} = A_0\sum_{i=0}^{m} B^i\,\varepsilon_{t+m-i} \tag{3}$$

The constraint of some variables implies the restrictions of on IRF (right hand side of the equation).

Doan et al (1984) shows that these constraints can be written as the following form

$$r = R\varepsilon \tag{4}$$

where vector r $(K\times m)\times 1$ is the path for constrained variables minus the unconditional forecast (K is the number of the constrained variables). R is a matrix [$(K\times m)$ $\times(N\times m)$]. The elements of R matrix are IRF of the constrained variables.

Doan et al (1984) solve the above equation by least square method:

$$\hat{\varepsilon} = (R'R)^{-1}R'r \tag{5}$$

and substitute back to the iterated equation (2).