The polynomial lags and mixed frequencies

The basic model is

$$y_t = \sum_{i=0}^{q} \beta_i x_{t-i} + e_t \tag{1}$$

where we can think of y_t as being a variable at quarterly frequency and x_t as being a variable at monthly frequency.

We want to approximate the parameters with a polynomial expression :

$$\beta_i = a_0 + a_1 i + a_2 i^2 + ... + a_p i^p \tag{2}$$

For simplicity assume that q=3 and p=2. This gives

$$y_t = \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \beta_3 x_{t-3} + e_t \tag{3}$$

Inserting the polynomial expression for β_i would give.

$$y_t = (a_0+a_1 0+a_2 0^2)x_t+(a_0+a_1 1+a_2 1^2)x_{t-1}+(a_0+a_1 2+a_2 2^2)x_{t-2}+(a_0+a_1 3+a_2 3^2)x_{t-3}+e_t \tag{4}$$

The right-hand side can now be written in matrix form as:

$$\begin{pmatrix} x_t & x_{t-1} & x_{t-2} & x_{t-3} \end{pmatrix} \begin{pmatrix} a_0 + a_1 0 + a_2 0^2 \\ a_0 + a_1 1 + a_2 1^2 \\ a_0 + a_1 2 + a_2 2^2 \\ a_0 + a_1 3 + a_2 3^2 \end{pmatrix} \tag{5}$$

Breaking out the a_i:s, one get:

$$\begin{pmatrix} x_t & x_{t-1} & x_{t-2} & x_{t-3} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} \tag{6}$$

The matrix H is of dimensions $(q + 1) \times (p + 1)$

A very similar derivation can be found in Intriligator (1978) Econometric models, techniques, and applications, page 183, equation 6.7.28. Also, in the documentation to the MiDaS Matlab toolbox the same matrix occurs (see equation 2.10 in the documentation for version 2.1 as of August 3, 2016). The documentation to EViews 9.5, Equation 28.6, also indicates that this is the weighting used in the estimation.

However, in the actual estimations of the Almon/PDL MiDaS model in the Matlab toolbox for MiDaS and in EViews 9.5, the weighting matrix is somewhat different. The weighting matrix used is

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{pmatrix} \tag{7}$$

A heuristic analysis of the consequences would imply that the curvature of the estimated β_i coefficients differ when using the different weighting matrices. The exact matrix that is used could perhaps, albeit not necessarily, be of lesser importance. It would be nice, however, to know the exact weighting scheme employed in the estimation.