4. The dynamic correlation-coefficient model

The multivariate GARCH model proposed by Engle (2002), which is used to estimate dynamic conditional correlations (DCC) in this paper, has three advantages over other estimation methods.[14] First, the DCC-GARCH model estimates correlation coefficients of the standardized residuals and thus accounts for heteroskedasticity directly. Second, the model allows us to include additional explanatory variables in the mean equation to measure a common factor. In this connection, we include U.S. stock returns as an exogenous global factor, rather than using the source of contagion (e.g., stock returns in Thailand) as an independent variable. Third, the multivariate GARCH model can be used to examine multiple asset returns without adding too many parameters.[15] The parsimonious parameter setting permits us to deal with up to 45 pair-wise correlation-coefficient series in a single representation. The resulting estimates of time-varying correlation coefficients provide us with dynamic trajectories of correlation behavior for national stock-index returns in a multivariate setting. This information enables us to analyze the correlation behavior when there are multiple regime shifts in response to shocks, crises, and credit-rating changes.

To start with, we specify the return equation as:

$$r_t = \gamma_0 + \gamma_1 r_{t-1} + \gamma_2 r_{t-1}^{\text{U.S.}} + \varepsilon_t, \tag{1}$$

where $r_t = (r_{1,t}, r_{2,t}, \ldots, r_{n,t})'$, $n = 10$; $\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \ldots, \varepsilon_{n,t})'$; and $\varepsilon_t | \Im_{t-1} \sim N(0, H_t)$.

Following the conventional approach, an AR(1) term and the one-day lagged U.S. stock return are included in the mean equation. The AR(1) is used to account for the autocorrelation of stock returns, which was found in almost all the markets under investigation, as reported in Table 1. The lagged U.S. stock returns have often been used to account for a global factor (Dungey et al., 2003).[16] The inclusion of the lagged U.S. stock returns is also based on the empirical finding that U.S. stock returns play an important role in determining stock returns in Asian countries and that Asian stock returns have no significant dynamic effect on U.S. stock returns. Next, we specify a multivariate conditional variance as:

$$H_t = D_t R_t D_t, \tag{2}$$

where D_t is the $(n \times n)$ diagonal matrix of time-varying standard deviations from univariate GARCH models with $\sqrt{h_{ii,t}}$ on the ith diagonal, $i = 1,2,\ldots,n$; R_t is the $(n \times n)$ time-varying correlation matrix. The DCC model proposed by Engle (2002) involves two-stage estimation of the conditional covariance matrix H_t. In the first stage, univariate volatility models are fitted for each of the stock returns and estimates of $\sqrt{h_{ii,t}}$ are obtained. In the second stage, stock-return residuals are transformed by their estimated standard deviations from the first stage. That is, $u_{i,t} = \varepsilon_{i,t}/\sqrt{h_{ii,t}}$, where $u_{i,t}$ is then used to estimate the parameters of the conditional correlation. The evolution of the correlation in the DCC model is given by:

$$Q_t = (1 - \alpha - \beta)\overline{Q} + \alpha u_{t-1} u'_{t-1} + \beta Q_{t-1}, \tag{3}$$

where $Q_t = (q_{ij,t})$ is the $n \times n$ time-varying covariance matrix of u_t, $\overline{Q} = E[u_t u'_t]$ is the $n \times n$ unconditional variance matrix of u_t, and α and β are nonnegative scalar parameters satisfying $(\alpha + \beta) < 1$.[17] Since Q_t does not generally have ones on the diagonal, we scale it to obtain a proper correlation matrix R_t. Thus,

$$R_t = (\text{diag}(Q_t))^{-1/2} Q_t (\text{diag}(Q_t))^{-1/2}, \tag{4}$$

where $(\text{diag}(Q_t))^{-1/2} = \text{diag}(1/\sqrt{q_{11,t}}, \ldots, 1/\sqrt{q_{nn,t}})$.

Now R_t in Eq. (4) is a correlation matrix with ones on the diagonal and off-diagonal elements less than one in absolute value, as long as Q_t is positive definite.[18] A typical element of R_t is of the form:

$$\rho_{ij,t} = q_{ij,t}/\sqrt{q_{ii,t} q_{jj,t}}, \quad i,j = 1,2,\ldots,n, \text{ and } i \neq j. \tag{5}$$

Expressing the correlation coefficient in a bivariate case, we have:

$$\rho_{12,t} = \frac{(1-\alpha-\beta)\overline{q}_{12} + \alpha u_{1,t-1} u_{2,t-1} + \beta q_{12,t-1}}{\sqrt{\left[(1-\alpha-\beta)\overline{q}_{11} + \alpha u_{1,t-1}^2 + \beta q_{11,t-1}\right]}\sqrt{\left[(1-\alpha-\beta)\overline{q}_{22} + \alpha u_{2,t-1}^2 + \beta q_{22,t-1}\right]}}. \tag{6}$$

As proposed by Engle (2002), the DCC model can be estimated by using a two-stage approach to maximize the log-likelihood function. Let θ denote the parameters in D_t, and ϕ the parameters in R_t, then the log-likelihood fund is:

$$l_t(\theta, \phi) = \left[-\frac{1}{2}\sum_{t=1}^{T} \left(n \log(2\pi) + \log|D_t|^2 + \varepsilon'_t D_t^{-2} \varepsilon_t \right) \right] + \left[-\frac{1}{2}\sum_{t=1}^{T} \left(\log|R_t| + u'_t R_t^{-1} u_t - u'_t u_t \right) \right]. \tag{7}$$

The first part of the likelihood function in Eq. (7) is volatility, which is the sum of individual GARCH likelihoods. The log-likelihood function can be maximized in the first stage over the parameters in D_t. Given the estimated parameters in the first stage, the correlation component of the likelihood function in the second stage (the second part of Eq. (7)) can be maximized to estimate correlation coefficients.

5. Evidence from dynamic correlations for the hardest-hit country group

5.1. Estimates of the model

Table 3 reports the estimates of the return and conditional variance equations. The AR(1) term in the mean equation is significantly positive for Thailand, Indonesia, Malaysia, Philippines, and Singapore, while it is significantly negative for Hong Kong and Japan. This finding is in agreement with the evidence in the literature in that the AR(1) is positive in emerging markets due to price friction or partial adjustment and that AR(1) is negative as the presence of positive feedback trading in advanced markets (Antoniou et al., 2005). However, AR(1) is not significant for Korea, Taiwan, and the United States. Consistent with most studies on Asian markets (Dungey et al., 2003), the effect of U.S. stock returns on Asian stock returns is, on average, highly significant and consistently large in magnitude, ranging from 0.155 (Indonesia) to 0.474 (Hong Kong). The coefficients for the lagged variance and shock-squared terms in the variance equation are highly significant, which is consistent with time-varying volatility and justifies the appropriateness of the GARCH(1,1) specification. Note that the sum of the estimated coefficients (see last column) in the variance equation $(a+b)$ is close to unity for all of the cases, implying that the volatility displays a highly persistent fashion.

Table 3
Estimation results from the DCC-GARCH model

	Return equations			Variance equations			
	γ_0	γ_1	γ_2	c	a	b	Persistence
TH	0.0448*	0.057***	0.228***	0.0615***	0.878***	0.109***	0.987
	(1.756)	(4.173)	(8.733)	(4.979)	(88.771)	(12.057)	
IN	0.0162	0.218***	0.155***	0.0137***	0.894***	0.117***	1.011
	(0.972)	(15.163)	(8.778)	(4.333)	(131.86)	(13.279)	
MA	0.0551***	0.129***	0.218***	0.0256***	0.892***	0.099***	0.991
	(3.224)	(9.856)	(14.090)	(5.817)	(117.59)	(13.084)	
KO	0.0145	0.001	0.324***	0.0454***	0.908***	0.082***	0.990
	(0.498)	(0.036)	(12.374)	(4.165)	(79.678)	(8.038)	
HK	0.0885***	0.030***	0.474***	0.0363***	0.926***	0.058***	0.984
	(4.532)	(2.568)	(23.344)	(6.018)	(160.23)	(13.712)	
JP	0.0005	0.046***	0.360***	0.0488***	0.899***	0.0798***	0.978
	(0.023)	(3.294)	(18.270)	(7.457)	(123.41)	(13.332)	
PH	0.0289	0.157***	0.282***	0.0582***	0.889***	0.0948***	0.983
	(1.165)	(10.703)	(11.773)	(5.359)	(97.069)	(11.975)	
SG	0.0457***	0.049***	0.330***	0.0316***	0.910***	0.071***	0.981
	(3.301)	(4.073)	(18.451)	(5.219)	(85.734)	(8.789)	
TW	0.0337	0.015	0.264***	0.0607***	0.917***	0.066***	0.983
	(1.124)	(1.183)	(9.090)	(5.601)	(105.89)	(9.545)	
U.S.	0.0559***	0.015		0.0047***	0.943***	0.055***	0.998
	(3.568)	(0.979)		(3.434)	(151.25)	(8.624)	

Notes: See notes in Table 2A. U.S. represents U.S. stock returns. The estimates of the mean-reverting process are α 0.006 (7.278) and β 0.989 (480.292). The persistence level of the variance is calculated as the summation of the coefficients in the variance equations $(a+b)$. The t-statistics are in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% levels with critical values of 2.58, 1.96, and 1.65, respectively.
Return equations: $r_t \quad \gamma_0 + \gamma_1 r_{t\ 1} + \gamma_2 r^{U.S.}_{t\ 1} + \varepsilon_t$, where $r_t \quad (r_{1,t}, r_{2,t}, \ldots, r_{10,t})'$, $\varepsilon_t \quad (\varepsilon_{1,t}, \varepsilon_{2,t}, \cdots, \varepsilon_{10,t})'$, $\varepsilon_t | I_{t\ 1} \sim N(0, H_t)$.
Variance equations: $h_{ii,t} \quad c_i + a_i h_{ii,t\ 1} + b_i \varepsilon^2_{i,t\ 1} \quad i \quad 1, 2, \ldots, 10$.

An advantage of using this model, as it stands, is the fact that all possible pair-wise correlation coefficients (45) for the 10 index returns in the sample can be estimated in a single-system equation.[19] To simplify the presentation and reduce unnecessary parameterizations in calculation, we examine the dynamic patterns of correlation changes by focusing on the hardest-hit markets, including Thailand, Indonesia, Malaysia, the Philippines, Korea, and Hong Kong.[20]