very helpful numerically.

2 The models

2.1 The APARCH Family

Most of the models `gig` can handle can be thought of as special cases of the Asymmetric Power ARCH (APARCH) model, introduced by Ding *et al.* (1993). This model is able to accommodate asymmetric effects and power transformations of the variance. Its specification for the conditional variance is the following:

$$\sigma_t^\delta = \omega' z_t + \sum_{i=1}^{q} \alpha_i (|u_{t-i}| - \gamma_i u_{t-i})^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta \tag{5}$$

where $\sigma_t \equiv \sqrt{h_t}$, the parameter δ (assumed positive, but typically ranging between 1 and 2) performs a Box-Cox transformation and γ captures the asymmetric effects. Special values of the parameters give rise to the special cases enumerated in Table 2.

MODEL	AUTHOR	CONSTRAINTS
ARCH	Engle (1982)	$\delta = 2$, $\gamma_i = 0$ for $i = 1, \ldots, q$ and $\beta_j = 0$ for $j = 1, \ldots, p$
GARCH	Bollerslev (1986)	$\delta = 2$, $\gamma_i = 0$ $i = 1, \ldots, q$
Taylor/Schwert GARCH	Taylor (1986) and Schwert (1990)	$\delta = 1$, $\gamma_i = 0$ $i = 1, \ldots, q$
GJR	Glosten *et al.* (1993)	$\delta = 2$
TARCH	Zakoian (1994)	$\delta = 1$
NARCH	Higgins and A. (1992)	$\gamma_i = 0$ for $i = 1, \ldots, p$ and $\beta_j = 0$ for $j = 1, \ldots, p$

Table 2: APARCH nested sub-models

The GJR model is given a special treatment in `gig`. Other software packages adopt a different, albeit equivalent, parametrisation for the same model (some programs even call it by some other name). What `gig` considers to be the GJR model

$$\sigma_t^2 = \omega' z_t + \sum_{i=1}^{q} \alpha_i (|u_{t-i}| - \gamma_i u_{t-i})^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{6}$$

is sometimes reparametrised as

$$\sigma_t^2 = \delta' z_t + \sum_{i=1}^{q} \left(\alpha_i u_{t-i}^2 + \gamma_i d_{t-i} u_{t-i}^2\right) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{7}$$