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INTERRELATED FACTOR DEMANDS FOR MANUFACTURING:
A Dynamic Translog Cost Function Approach

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In this paper we report the results of estimating a joint model for labour, capital and energy using the transcendental logarithmic cost function of Christensen, Jorgenson and Lau (1973). This allows us to determine both the own price and cross elasticities of substitution between labour, capital and energy. We combine a model of factor shares with a dynamic adjustment process for each factor. We also impose on the cross equation restrictions necessary to provide additivity, homogeneity and symmetry. Since the covariance matrix in systems of share relationships is singular we follow the suggestions of Anderson and Blundell (1982) and employ a multivariate error corrrection model. The results suggest that additivity, symmetry and homogeneity restrictions do hold and that there are short run increasing returns to scale.

1. Introduction

In this paper we report the results of estimating a joint model for labour, capital and energy using the transcendental logarithmic cost function of Christensen, Jorgenson and Lau (1973). This allows us to determine both the own price and cross elasticities of substitution between labour, capital and energy. We are then in a position to establish how changes in relative factor prices will cause substitution into and out of particular factors. We can also consider the extent to which energy and capital are complements or substitutes – an issue which has been the subject of controversy since the work of Berndt and Wood (1979).

Our work raises a number of issues, and technical difficulties. Economic theories of production require that certain restrictions concerning the effects of factor prices on the use of labour, capital and energy are accepted. Since the use of a translog cost function involves estimating share equations for labour, capital and energy in total cost, this means that additivity, homogeneity and symmetry must hold. However, a number of empirical studies, both of factor and commodity demand systems, have suggested that impos-

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ing these types of restriction on the data induces serial correlation which suggests that the model is misspecified. One response to this has been to suggest that the serial correlation may be because the dynamics have been misspecified rather than because the restrictions implied by economic theory are invalid. It may be that the restrictions hold in long run equilibrium but not during the period of short term adjustment.

Indeed there is a large empirical literature on dynamic, and sometimes interrelated, factor demand models.[1] The dynamic adjustment processes which these models rely upon can be rationalised in terms of the costs of adjustment models of Eisner and Strotz (1963). Lucas (1967), Mortensen (1973) and Treadway (1969). The multivariate flexible accelerator model has been the basis of many empirical studies and recently has been given a firmer theoretical underpinning by Epstein and Denny (1983).

In this paper we combine a model of factor shares with a dynamic adjustment process for each factor. We also impose the cross equation restrictions necessary to provide additivity, homogeneity and symmetry of the cost function. Since the covariance matrix in systems of share relationships is singular we follow the suggestions of Anderson and Blundell (1982) and employ a multivariate error correction model. In contrast to many other empirical studies we do not assume that any of the factors are quasi-fixed, so each factor is allowed to adjust at its own rate.

In section 2 we derive the share equations from a model in which firms minimise total cost subject to output and factor prices. We also show how this static framework can be embedded in an adjustment process in which the cross equation restrictions hold only in the long run, while in the short run capital, labour and energy usage adjust towards desired levels.

In section 3 we report the results of estimating this model. Various tests of the validity of the restrictions are provided. Finally, we consider how the model can be developed and how it can be extended to allow for intertemporal cost minimisation.

2. The production sector

In this section we develop a model of production and the demand for capital, labour and energy using a translog cost function approach. We have assumed that we can use a twice differentiable aggregate production function which relates the flow of gross output, X, of the manufacturing sector to three inputs, labour, L, capital K, and energy, E.[2] There are constant returns to scale, and technical progress is Hicks-neutral, or Harrod-neutral.

[1]See Nadiri and Rosen (1969), Coen and Hickman (1970) as well as Sargent (1978), Kennen (1979) and Berndt and Morrison (1981).

[2]Material has been excluded from the analysis because of data inadequacies. For this not to matter we require that materials are at least weakly separable from labour, capital and energy.

There exists a cost function which corresponds to this production relationship. We assume that this takes a particular functional form which allows us to test a number of hypotheses concerning the behaviour of the factors which produce any level of output. It also allows us to determine the degree of substitutability or complementarity between factors of production. Our work differs from many previous studies of factor demands because we distinguish between long-run, equilibrium steady state paths and short run dynamics and we do not impose any particular type of dynamic adjustment.[3] Subject to stable steady-state paths for the exogenous variables, we examine the steady-state substitution possibilities between factors while allowing the data to determine the short-run relative price responses. Indeed we also allow for the possibility of short-run increasing or decreasing returns to scale.[4] The equilibrium cost function is

$$\begin{aligned}\ln TC = {} & \alpha_0 + \ln Y + \alpha_k \ln P_k + \alpha_l \ln P_l + \alpha_e P_e + 1/2\sigma_{kk}(\ln P_k)^2 \\ & + \sigma_{kl} \ln P_k \ln P_l + \sigma_{ke} \ln P_k \ln P_e + 1/2\sigma_{ll}(\ln P_l)^2 \\ & + \sigma_{lk} \ln P_l \ln P_k + \sigma_{le} \ln P_l \ln P_e + 1/2\sigma_{ee}(\ln P_e)^2 \\ & + \sigma_{ek} \ln P_e \ln P_k + \sigma_{el} \ln P_e \ln P_l + \sigma_{lT} \ln P_l \cdot T + \sigma_{kT} \ln P_k \cdot T \\ & + \sigma_{eT} \ln P_e \cdot T. \end{aligned} \tag{1}$$

This form of cost function places no restrictions on the partial elasticities of substitution between factors and can be interpreted as a quadratic approximation to some general twice differentiable cost function [Berndt and Wood (1975)]. In (1) P_i is the user cost of the utilisation of the ith factor, Y is gross output, TC is total cost and T is a time trend. This function has equilibrium constant returns to scale. We can also apply symmetry, which implies $\sigma_{ij} = \sigma_{ji}$, and linear homogeneity which implies

$$\sum_i \alpha_i = 1 \quad \text{for} \quad i = L, K, E. \tag{2}$$

Hicks' neutral technical progress is given by $\delta_{iT} = 0$, *for* $i = L, K, E$. Harrod neutral technical progress by

[3] A related study to ours which used a cost function approach on UK data is that of Asteraki (1984). However, he was unable to impose cross-equation coefficient restrictions without also constraining the speed of adjustment to be the same for all three factors. We find that the data rejects this assumption.

[4] Berndt and Morrison (1981).

$$\sum_i \delta_{iT}=0 \quad \text{for} \quad i=L,K,E. \tag{3}$$

If we assume that the constituent firms of the sector minimise costs, treating factor prices and the level of output as conditionally predetermined, then

$$\frac{\partial \ln TC}{\partial \ln P_i}=\frac{\partial TC}{\partial P_i}\frac{P_i}{TC}=\alpha_i+\sum\sigma_{ij}\ln P_i+\delta_{iT}\cdot T. \tag{4}$$

Using Shephard's Lemma – the level of usage of any factor is equal to its marginal own price effect on total cost – implies input demand equations in the shares of total cost as

$$s_k=P_kK/TC=\alpha_k+\sigma_{kk}\ln P_k+\sigma_{kl}\ln P_l+\sigma_{ke}\ln P_e=\sigma_kT, \tag{5}$$

$$s_l=P_lL/TC=\alpha_l+\sigma_{ll}\ln P_l+\sigma_{lk}\ln P_k+\sigma_{le}\ln P_e=\sigma_lT, \tag{6}$$

$$s_e=P_eE/TC=\alpha_e+\sigma_{ee}\ln P_e+\sigma_{ek}\ln P_k+\sigma_{el}\ln P_l=\sigma_eT, \tag{7}$$

where total cost is defined as

$$TC=P_kK+P_lL+P_eE. \tag{8}$$

In the short run we might expect different speeds of adjustment for different factors, in particular slow adjustment may induce increasing or decreasing returns to scale. To capture this the cost function can have terms added to it

$$+\beta_{ly}\ln P_l\cdot\Delta\ln Y+\beta_{ky}\ln P_k\cdot\Delta\ln Y+\beta_{ey}\ln P_e\cdot\Delta\ln Y. \tag{9}$$

These terms therefore appear in the individual share equations as $+\beta_{ly}\Delta\ln Y$, $\beta_{ky}\Delta\ln Y$, and $\beta_{ey}\Delta\ln Y$, in the SK, SL and SE equations respectively.

These factor demand equations form the basis for our estimates. For later reference we define the Allen partial elasticities of substitution between factors i and j as[5]

$$\begin{aligned}\gamma_{ii}&=(\sigma_{ii}-s_i)/s_i^2,\\ \gamma_{ij}&=\sigma_{ij}(s_is_j)^{-1}-1.\end{aligned} \tag{10}$$

[5]Berndt and Wood (1975), Uzawa (1962). Strictly speaking the Allen definition of the cross elasticities is not the same as Joan Robinson's definition of the elasticity of the substitution since Allen's are price elasticities weighted by the factor share. See Kang and Brown (1981) for a discussion and some alternative definitions of elasticities.

These elasticities of substitution are clearly dependent upon the level of the shares at the point of time they are measured. Conventional price elasticities of demand for factors will vary between factors even when symmetry is imposed since they are defined as

$$E_{ij}=s_j\gamma_{ij}, \qquad i,j=K,L,E. \tag{11}$$

As Berndt and Wood point out, since the cost shares sum to unity, the covariance matrix is singular. Anderson and Blundell (1982, 1983) have suggested one approach to estimating share equations where homogeneity and symmetry can be imposed on factor prices in the long run while allowing disequilibrium adjustment in the short run. This type of approach has particular attractions because it allows us – in principle – to reconcile long run system restrictions with the observable data generation process. Suppose that the long-run share relationships given by (5) to (7) are written as

$$s_t=\Pi x_t, \tag{12}$$

where $s_t'=(s_l,s_k,s_e)$ and $x_t'=(l,w,q,p_e,T)$. w is the user cost of labour, q the user cost of capital and p_e the cost of energy, T is a time trend. The homogeneity, additivity and symmetry restrictions on the factor prices take the form

$$\sum_{i=1}^{3}\pi_{il}=1; \qquad \sum\alpha=1, \qquad \text{(homogeneity)}$$

$$\pi_{ij}=\pi_{ji} \quad \text{for} \quad i=1,3;\ j=2,4;\ \rho_{ij}=\pi_{ji}, \qquad \text{(symmetry)}$$

$$\sum_{i=1}^{3}\pi_{ij}=0 \quad \text{for} \quad j=2,4, \qquad \text{(additivity)}$$

$$\sum_{i=1}^{3}\pi_{i4}=0, \qquad \text{(Harrod Neutrality)}$$

$$\pi_{i4}=0 \quad \text{for} \quad i=1,3. \qquad \text{(Hicks Neutrality)}$$

A dynamic form of (2.12) can be written

$$C(L)s_t=D(L)x_t+\varepsilon_t, \tag{13}$$

where $C(L)$ and $D(L)$ are polynomials in the lag operator L,

$$C(L)=C_0+C_1+C_2L^2+\cdots+C_pL^p,$$

$$D(L)=D_0+D_1+D_2L^2+\cdots+D_qL^q.$$

ε_t is an iid vector of random disturbances.

If a stable solution exists to this model then the long run solution is

$$\Pi=\left[\sum_{j=0}^{p}C_j\right]^{-1}\left[\sum_{i=0}^{q}D_i\right]. \tag{14}$$

The problem lies in using the general dynamic model given by (13) to impose the required restrictions. Anderson and Blundell (1982) suggest that the observationally equivalent model

$$\Delta s_t=-A(L)\Delta s_t+B(L)\Delta x_t^*-K[s_{t-p}-\Pi x_{t-q}]+\varepsilon_t \tag{15}$$

be used, where

$$A(L)=\sum_{i=1}^{p-1}\left[\sum_{j=0}^{i}C_j\right]L^i, \qquad p>1;\text{ null otherwise}$$

$$B(L)=\sum_{i=1}^{q-1}\left[\sum_{j=0}^{i}D_j^*\right]L^i, \qquad q\geqq 1;\text{ where } D^* \text{ is } D \text{ with the first column deleted.}$$

$$K=I+C_1+C_2+\cdots+C_p,$$

and x^* is x with the first element (the constant term) deleted.

The equation system (15) cannot be estimated directly as it has the property that any one of the n equations can be expressed as a linear combination of the other $n-1$. This problem of redundancy of one the shares applies equally to the vector of lagged dependent variables. One way of overcoming the singularity of (15) is to delete one of the collinear share equations. If subscript n denotes that a matrix had had its nth row deleted, and superscript n indicates a $(n\times n-1)$ dimensional matrix, we can rewrite (15) as

$$\Delta s_t=-A^n(L)\Delta s_{nt}+B(L)\Delta x_t^*-K^n[s_{nt-p}-\Pi_n x_{t-q}]+\varepsilon_t, \tag{16}$$

where s = vector of cost shares, x = vector of exogenous variables (factor prices, output), s_n = vector s with row n deleted, A^n = matrix A with column n deleted.

The coefficient matrices A^n, K^n now contain elements which are linear combinations of the original coefficients in A and K. Anderson and Blundell (1982) show that it is not possible to identify the elements of (16). Using the adding-up restrictions, we require that each of the columns of A^n and K^n sum to zero. As our interest is in the long-run solution of the model, (12), the fact that we can identify the elements of π using the adding-up restrictions is sufficient.

Interpreting the above model is easier if we look at the particular form of the factor demand model we have estimated for manufacturing. The model comprises the factor shares for labour, capital and energy dependent on the wage rate, the user cost of capital and the price of energy plus a time trend to pick up technical progress and output to detect short run decreasing or increasing returns to scale.

$$\begin{bmatrix} \Delta sl_t \\ \Delta sk_t \\ \Delta se_t \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} \Delta sl_{t-1} \\ \Delta sk_{t-1} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \end{bmatrix} \begin{bmatrix} \Delta w \\ \Delta_q \\ \Delta p_e \\ \Delta y \end{bmatrix}$$
$$- \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \\ k_{31} & k_{32} \end{bmatrix} \begin{bmatrix} sl_{t-1} \\ sk_{t-1} \end{bmatrix} - \begin{bmatrix} \pi_{10} & \pi_{11} & \pi_{12} & \pi_{13} & \pi_{14} \\ \pi_{20} & \pi_{21} & \pi_{22} & \pi_{23} & \pi_{24} \\ \pi_{30} & \pi_{31} & \pi_{32} & \pi_{33} & \pi_{34} \end{bmatrix} \begin{bmatrix} 1 \\ w \\ q \\ p_e \\ T \end{bmatrix}$$
$$+ \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \tag{17}$$

where

$$\alpha_{31} = -(\alpha_{11} + \alpha_{21}), \quad k_{31} = -(k_{11} + k_{21}),$$

$$\alpha_{32} = -(\alpha_{12} + \alpha_{22}), \quad k_{32} = -(k_{12} + k_{22}).$$

The cost-minimizing representative firm is assumed to be able to adjust all three factors fully in the long run. Therefore the cost function appropriate to the long-run position of the firm is given by the static model of Berndt and Wood (1975); eq. 1.

Since there are costs to be incurred by changing the level of usage of the factors of production, a common approach has been to distinguish between quasi-fixed and flexible factors. We have, however chosen to treat all factors as partially flexible and to use dynamic models to establish how factor usage is adjusted over time. We can then distinguish between short-run costs which are given by the relative prices of the flexible factors and adjustment costs of the quasi-fixed factors, and the long-run costs given by the relative prices of all factors. In many studies, adjustment costs are assumed to be quadratic so that demand equations for quasi-fixed factors are generated with one lagged dependent variable. As Nickell (1984) has highlighted, quadratic adjustment costs will not generate the second order autoregressive process in labour usage which is observed empirically.

If the model of adjustment costs is to fit the data, higher order functions may be necessary. Econometric examination of these functional forms has been limited so far. We have chosen to rely on unrestricted dynamic responses generated by the data. If, as is to be expected, capital and labour are much more sluggish in their response to exogenous shocks as compared to the usage of energy, then this will be uncovered by the nature of the dynamic process estimated.

The long run solutions to the share equations are of the form of eqs (5) to (7). These solutions apply when all factors have adjusted fully to any output or relative price disturbance and, therefore, represent the long-run cost-minimising choice without adjustment costs. Given that we have theoretical priors concerning the technology which links factor demands in equilibrium we can apply restrictions across equations at this stage.

3. Discussion of results

In this section we describe our attempts to apply the method of section 2 to the demand for factors of production by the manufacturing sector of the UK. The flexible approach to dynamics within a well defined equilibrium framework allows us to examine some issues highlighted by the recent literature on factor demands though this is not our primary concern. The possibility that there are increasing returns to scale in the short run while in the long run returns to scale are constant has been considered by Morrison and Berndt (1981). They utilise a model of quasi-fixed capital and derive short run increasing returns to scale as a natural outcome of the short run cost minimising behaviour of the firm. In our formulation of the cost minimising problem adjustment speeds are endogenous and can be interpreted flexibly since we do not specify explicit adjustment cost functions. Indeed we do not require any factor to be predetermined so the possibility of SRIRL is left as an empirical issue. The crucial parameters are on the terms Δy_t in the estimated share equations for labour, i.e., α_8 and α_9 in eq. (12). Our results suggest that there are significant short run increasing returns to scale in labour utilisation.

The most interesting aspect of these results is the significance and strength of the effect of the user cost of capital on the demand for capital, labour and energy, a result which contrasts sharply with other studies [Bean (1983), Beenstock et al. (1984)]. This implies that shocks to the economy that change factor prices will be transmitted by factor substitution as well as through the demand channels which are normally emphasised.

3.1. Data

We have examined the models of section 2 using data on the manufacturing sector of the UK for the period 1968q3 to 1979q3. The choice of this period was in part determined by our recognition of the inadequacy of the official data on the physical capital stock. There is widespread evidence of writing-off of capital following the recession of 1979–80 which makes the official measure, based on historic cost measurement, unlikely to represent the economic capital stock. We think that this problem is most acute for the post-1979 period. We shall attempt, however, to use our results to cast light on behaviour over that period. The choice of a three-factor cost function was in part determined by the unavailability of data on material usage in manufacturing. We are assuming implicitly that this decision is separable from the rest. In future work we intend to experiment with data on total UK imports of raw materials rather than calculate material usage as a residual from gross output and other factor costs. The data used in compiling the results below is described more fully in an appendix.

The cost of capital fluctuates much more widely than the cost of labour, while energy costs are closely correlated with the jumps in oil prices in 1973 and 1979. The cost of capital varies most because of variations in the (expected) rate of inflation. One potential drawback to the measure of the cost of capital we have used is that it ignores the effect of 'tax exhaustion'. Under the UK corporation tax system – at least until 1984 – there were a variety of capital allowances that firms could claim against their tax payments. However, a number of firms were not actually paying taxes because the profits they were making were insufficient to cover the value of claimable allowances. This means that the effective real cost of capital to a tax exhausted firm is higher than for a firm which has taxable profits against which investment costs can be offset. There is relatively little empirical evidence on the degree of importance of tax exhaustion, though Holly, Longbottom and Haque (1985) have found that tax exhaustion matters empirically in a 'Q' type model of investment. Thus the real cost of capital we employ makes allowance for both the corporation tax rate and investment allowances but ignores tax exhaustion so that our aggregate measure of the cost of capital may be biased downwards.

3.2. Estimation

In section 2 a general dynamic specification was described. It is clear, however, that the small number of degrees of freedom makes a fully nested general to specific search impossible. It is important to note that for any nesting procedure the criteria for excluding particular lags of variables is now based on the effect on the whole model rather on single equations. We started, initially, with up to two lags in each variable, but found that restricting the second lags on the conditionally exogenous variables to zero was an acceptable restriction on the model. The results of estimating system (17) are shown in table 1.

The results are shown in two parts. The first set are estimates of the steady-state coefficients of eq. (12) with homogeneity, symmetry and additivity imposed. Technical progress is Harrod neutral with continuous capital deepening and substitution out of labour. The actual rate of capital deepening and substitution out of labour cannot be inferred directly from these estimates since they refer to the effect on the factor shares. To determine the effects on actual factor use we need to also take into account how the real wages change in response to increases in productivity and this cannot be done without a more complete specification of a model of the labour market. Where coefficients appear without standard errors they have been inferred from the estimates of the other equations. Although additivity, symmetry and homogeneity have been imposed on the steady-state solutions, not all these restrictions are imposed in the short run. Estimated factor shares in total cost will always sum to one, giving the adding-up restrictions outlined in section 2. During the adjustment process we allow the effect of a change in one of the factor prices on each share to be asymmetrical.

Likelihood ratio tests were used to assess the validity of the restrictions imposed on the model and are shown in table 1. The test values are shown in the lower half of the table. In each case we report both the standard likelihood ratio statistic, ϕ, and the 5% asymptotic critical value for this χ^2 test, k_0. It is to be expected that the power of the test will be sensitive to the sample size, so we also provide small-sample corrected test statistics following Pudney (1981) and Sargan and Sylwestrowicz (1976).[6] The values of the test statistics provide a grid of possible outcomes we can use to assess the sensitivity of the tests to the effects of the size of the sample. Four restrictions are needed to impose additivity, symmetry and homogeneity. The uncorrected test statistics using the standard 5% χ^2 critical value do not

[6]The corrected value of ϕ is calculated as $\phi^* = \phi + nT(\log\{(nT-p_1)/(nT-p_0)\}$ where n is the number of equations, T the number of observations, and p_1 and p_0 the number of parameters in the unrestricted and restricted models respectively. The corrected values of the critical values exploit the relationship between the likelihood ratio and F statistics. $k_1 = 2nT\log(1 + dF^d_{T^*}/T^*)$, where $d = p_1 - p_0$, $T^* = nT - p_1$ and $F^d_{T^*}$ is the 5% critical value of the F distribution, and $k_2 = nT\log(1 + dF_{T_1}/T^*)$ where $T_1 = T - p_1/n$.

Table 1

Restricted form: Steady state

$$\begin{bmatrix} sl \\ \\ sk \\ \\ se \end{bmatrix} = \begin{bmatrix} 0.1791 & 0.1571 & -0.0981 & -0.0590 & -0.0009175 \\ (4.62) & (16.28) & & (7.23) & (7.23) \\ 0.772 & -0.0981 & 0.1059 & -0.0078 & 0.0009175 \\ (28.87) & (16.47) & (25.26) & & (7.23) \\ 0.0489 & -0.0590 & -0.0078 & 0.0668 & 0.0 \end{bmatrix} \begin{bmatrix} 1 \\ w \\ q \\ p_e \\ T \end{bmatrix}$$

Dynamics

$$\begin{bmatrix} \Delta sl_t \\ \\ \Delta sk_t \\ \\ \Delta se_t \end{bmatrix} = \begin{bmatrix} -0.056 & 0.0783 \\ (0.65) & (0.87) \\ 0.099 & 0.163 \\ (1.53) & (2.43) \\ & \end{bmatrix} \begin{bmatrix} \Delta sl_{t-1} \\ \\ \Delta sk_{t-1} \\ \\ \Delta se_{t-1} \end{bmatrix} - \begin{bmatrix} -0.070 & -0.6160 \\ (2.10) & (3.66) \\ 0.540 & 0.992 \\ (5.03) & (6.78) \\ & \end{bmatrix} \begin{bmatrix} sl_{t-1} \\ \\ sk_{t-1} \\ \\ se_{t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0.236 & -0.102 & -0.0527 & -0.0651 \\ (8.07) & (21.3) & (7.74) & (2.59) \\ -0.143 & 0.111 & -0.014 & -0.051 \\ (6.70) & (21.3) & (2.6) & 0.26) \\ & & & \end{bmatrix} \begin{bmatrix} \Delta w_t \\ \Delta q_t \\ \Delta p_{et} \\ \Delta y_t \\ \end{bmatrix}$$

Test statistics

Maximised value of the likelihood function = 428.60

Likelihood ratio tests

	k_0	k_1	k_2	ϕ	ϕ^*
Steady state	9.49	11.62	12.44	8.44	3.57
absence of x in steady state	5.99	7.31	7.71	3.44	1.03
1st order vector *AR*	9.49	11.77	12.64	4.62	0.29
2nd order vector *AR*	15.51	19.59	21.46	20.5	10.48

LR test of parameter constancy 31.78 (critical value $\chi^2(27) = 40.01$)

$DW\ (sl) = 2.14$ $\quad DW(sk) = 1.79$

LM tests of weak exogeneity:	Granger Causality:
w: 1.290	w: 5.13
q : 0.264	q : 43.17
e : 0.254	e : 29.71
x : 0.469	x: 3.08

$\chi^2(2)$ 5%: 5.99 $\quad F(4.36)$ 5%: 2.65

Partial elasticities of substitution (at sample means)

	L	*K*	*E*
w	−0.0478		
q	0.1492	−0.1531	
e	0.1795	0.4881	−0.2155

Eigenvalues of adjustment matrix

$0.469 \pm 0.265i$
-0.018
0.263

reject the restrictions. The maintained assumption is that there are long run constant returns to scale, and this allows us to use a particularly simple cost function and set of share equations. Non-constant returns to scale can be tested by excluding x from the share equations. The asymptotic ratio test for the exclusion of x from the steady state solution has a value well below the asymptotic 5% critical value, so we can reject the hypothesis that there are non-constant returns to scale.

Next we consider the restrictions we have imposed on the dynamic structure of the model. In the initial specification only first order lags in the factor prices and output and second order lags in the shares were significant. Using the approach of Breusch and Godfrey (1980) we calculated likelihood ratio tests for the significance of added vectors of lagged residuals to eq. (17). When we tested for first order serial correlation we were able to reject it. However, the test for a vector $AR(2)$ process was more marginal. The asymptotic test statistic ϕ suggests that second order autocorrelation is present. However, the small-sample adjusted statistic rejects the presence of $AR(2)$ errors even at the asymptotic 5% critical value. These results support the particular dynamic specification we have adopted and also highlights the importance of the sample size.

In the discussion earlier we reported that the estimation period has been confined to end in 1979(3). However, we tested for parameter constancy by including observations up to 1982(4). A generalisation of a Chow test due to Andersen and Mizon (1983) was used. The value reported in table 1 is well below the 5% critical value which is of the form $\chi^2(nT_2)$, where T_2 is the number of extra periods.

We have also assumed that all of the left hand side variables are exogenous. For consistent estimation and inference we require that the explanatory variables be weakly exogenous. In table 1 we present an LM test of this proposition. This test is due to Engle (1983) and is similar to those of Wu (1972) and Hausman (1969). The resulting small figures show that we cannot reject weak exogeneity for all the RHS variables. It should be noted, however, that this test has relatively low power. For forecasting and simulation, Engle, Hendry and Richard (1983) show that we require both weak exogeneity and Granger non-causality for the RHS variables. Given that we accept weak exogeneity we need to show that the factor prices and output cannot be explained by the lagged shares. In column 2, however, this hypothesis is soundly rejected. This suggests that we need to model the determinants of factor prices and output simultaneously with the share equations to take account of the interactions between factor usage and factor prices.

In the last section of table 1 we report the partial elasticities of substitution between factors generated by the long-run equilibrium form of the model. The general result that factors are substitutes is supported. Given that these elasticities are share-dependent and were calculated at sample means, they should be interpreted with caution. For instance the own-price elasticity of labour is not 'low' at -0.04 as suggested by Asteraki (1982) as conventional elasticities of factor usage are not the same as the partial elasticies of substitution in total cost.

3.3. Further developments

Our results indicate that there are significant effects of factor prices on the use of labour, capital and energy. They also suggest that labour, capital and energy are substitutes, even in the short run. These results are at variance with other estimates where energy and capital are found to be complements [Berndt and Wood (1975), Pindyck and Rotemberg (1983)]. The substitution effects between capital, labour and energy which we identify may be due to the fact that we have treated adjustment costs in a general way which is data coherent. This is in contrast to many previous papers where the analysis and estimates are essentially static. There are, however, some possible shortcomings of our analysis. First, our neglect of material usage may be biasing the estimation results for the cross elasticities.

Second, our model – although dynamic – is essentially backward-looking, and does not allow for intertemporal cost minimisation or for intertemporal substitutions. Pindyck and Rotemberg (1983) have shown how the problem can be set up in such a way that we can retain the flexibility of the translog functional form, while allowing firms to minimise the expected sum of discounted costs. This should be the basis for future work.

Appendix

Data definitions.

(a) User cost of labour (PL)

Average straight time earnings in manufacturing are adjusted for the average rate of tax on employers, thus

$$PL = AEMS * \left(1 + \frac{(YEC + TNIS)}{YWS}\right),$$

where	YEC	: Employers contributions	ET
	$TNIS$	: National Insurance surcharge	$FINS$
	YWS	: Total wage and salary bill	ET

Average straight time earnings are derived from the published average earnings series by allowing for an overtime premium of 30%, thus

$$AEM = AEMS * NHOURS + 1.3 * [AEMS * (HOURS - NHOURS)],$$

where	*AEM*	: Average earnings in manufacturing	*DEG*
	NHOURS	: Normal hours worked in manufacturing	*DEG*
	HOURS	: Hours worked in manufacturing	*DEG*

These indices (1980 = 100) can be converted into the published earnings per employee by the multiplicative constant 0.03471.

(*b*) *User cost of capital* (*PK*)

Calculations carried out following King (1974) and many others were as follows

$$P_k = \frac{PI(1-A)(\delta + R*(1-\tau) - 0.5*\pi)}{(1-\tau)}.$$

τ	= corporation tax rate,	*FINS*
PI	= Price index of manufacturing investment,	*ET*
A	= Present value of investment allowances	*BOE*
δ	= 1.75%/quarter	
$R*$	= $(RLB+2)/4$, quarterley interest rate, or $RL/4$	*FINS*
π	= autoregression of ΔPI on ΔPI_{t-1}, ΔPI_{t-2}	

$$\pi_k = 100 * (0.62515 \Delta PI_{t-1} + 0.37485 \Delta PI_{t-2} + \text{dummy variables})$$

where	*RLB*	: *MLR*/Clearing banks base rate	*FINS*
	RL	: Interest rate on undated consols	*FINS*

The results in the main part of the paper are generated using the data derived by Kelly and Owen (1985) using a similar formula.

(*c*) *User cost of energy* (*Pe*)

The implicit deflator for energy used by manufacturing published in *ENT*; 1980 = 100 index. This index can be converted into the published cost per therm by the multiplicative constant 0.3037114.

(d) Labour usage (L)

Employment in manufacturing, ,000 *ET.*

(e) Capital stock (K)

The capital stock series for manufacturing (£,000m, 1980 prices) is interpolated from the annual series provided by the CSO. This uses a straight-line retirements approach and includes data on capital leased by the manufacturing sector.

(f) Fuel usage (E)

Energy used by manufacturing (million therms) *DES.*

(g) Net output (X)

Index of net output in manufacturing *ET.*
This index can be converted into the published net output figure in £,000m by the multiplicative constant, 131.225.

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