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ORIGINAL PAPER

Tests for cointegration with two unknown regime shifts with an application to financial market integration

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Abstract It is widely agreed in empirical studies that allowing for potential structural change in economic processes is an important issue. In existing literature, tests for co-integration between time series data allow for one regime shift. This paper extends three residual-based test statistics for cointegration to the cases that take into account two possible regime shifts. The timing of each shift is unknown a priori and it is determined endogenously. The distributions of the tests are non-standard. We generate new critical values via simulation methods. The size and power properties of these test statistics are evaluated through Monte Carlo simulations, which show the tests have small size distortions and very good power properties. The test methods introduced in this paper are applied to determine whether the financial markets in the US and the UK are integrated.

Keywords Structural break · Cointegration · Size · Power · Monte Carlo simulations

JEL Classification C12 · C15 · C22 · C52 · G11 · G15

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1 Introduction

Testing for cointegration between variables with unit roots is an integral part of empirical time series analyses. A number of tests are available in the literature. Most of these tests are residual based and they are widely used due to their simplicity. However, these tests were introduced based on the assumption that the cointegrating vector remained the same during the period of study. There are many reasons to expect that the long-run relationship between the underlying variables might change (shifts in the cointegrating vector can occur). Structural changes can take place because of economic crises, technological shocks, changes in the economic actors' preferences and behaviour accordingly, policy and regime changes, and organizational or institutional evolution. This is more likely to be the case if the time span is long. In a detailed seminal study by Perron (1989) tests for unit roots are shown to have low power in the presence of structural breaks that are not taken into account. Perron (1989) suggested a modification of unit root tests that allow for one break with a predetermined timing. Tests for unit roots in the presence of breaks have been a topic of active research since Perron's paper. Several tests have been developed that automatically can take into account one or more than one structural breaks (see among others Banerjee et al. 1992; Zivot and Andrews 1992; Perron 1989; Bai and Perron 1998, 2003). It is shown by Gregory and Hansen (1996) that tests for cointegration have also low power in the presence of a regime shift that is not taken into account. Gregory and Hansen (1996) modify three residual based unit root tests that take into account one unknown regime shift. They suggest a procedure to choose the timing of a shift in the cointegrating vector based on the data. They furthermore provide new critical values for the ADF test for cointegration (as suggested by Engle and Granger 1987) and two tests known as Z_a and Z_t (as suggested by Phillips 1987). The current paper builds on the Gregory and Hansen (1996) tests for cointegration in the presence of one shift and by extending their work to allow for two regime shifts. We also provide new critical values for ADF, Z_a and Z_t tests under these circumstances. Again the timing of the breaks is not based on *a priori* information but rather upon the underlying data. All the Mote Carlo simulations are conducted by programming in Gauss. A consumer friendly programme procedure for applying the tests methods presented in this paper is available from the author upon request.

The rest of the paper is organized thus: Sect. 2 defines the model and three test statistics for cointegration in the presence of two unknown regime shifts. Section 3 presents the procedure that is used to produce asymptotic critical values for these test statistics. Section 4 evaluates the size and power properties of these test statistics. Section 5 gives an application. The last section provides conclusions.

2 The model for two regime shifts

Consider the following regression equation:

$$y_t = \alpha + \beta' x_t + u_t, \quad t = 1, 2, \ldots, n. \tag{1}$$

where y_t is the dependent variable; x_t is an m-dimensional vector of independent variables, α is the intercept term, β is a m-dimensional vector of slopes and t represents the time index. Equation (1) is the standard model that is usually applied to test for cointegration. According to Engle and Granger (1987) cointegration prevails if u_t is integrated of degree zero, $u_t \sim I(0)$, provided that $y_t \sim I(1)$ and $x_t \sim I(1)$.[1] Based on this idea to test for cointegration the variables are tested for unit roots. If a unit root exists in each variable in the model represented by Eq. (1) then the linear combination approximated by u_t is tested for one unit root. Three residual based test statistics, namely the augmented Dickey–Fuller (ADF) test (suggested by Engle and Granger 1987), and the Z_a and Z_t tests (suggested by Phillips 1987), are commonly used to detect whether there is cointegration or not. However, Gregory and Hansen (1996) showed that these test statistics for cointegration are misspecified if a structural shift has occurred during the period of study. They extended these test statistics to allow for one regime shift at an unknown time,[2] which is determined by the data. The authors generate new critical values. This paper extends the tests for cointegration to take into account the possibility of two structural shifts during the time period of study.

To account for the effect of two structural breaks on both the intercept and the slopes (two regime shifts), we generalize Eq. (1) to the following equation

$$y_t = \alpha_0 + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \beta_0' x_t + \beta_1' D_{1t} x_t + \beta_2' D_{2t} x_t + u_t, \tag{2}$$

where D_{1t} and D_{2t} are dummy variables defined as

$$D_{1t} = \begin{cases} 0 & \text{if } t \leq [n\tau_1] \\ 1 & \text{if } t > [n\tau_1] \end{cases}$$

and

$$D_{2t} = \begin{cases} 0 & \text{if } t \leq [n\tau_2] \\ 1 & \text{if } t > [n\tau_2] \end{cases}$$

with the unknown parameters $\tau_1 \in (0, 1)$ and $\tau_2 \in (0, 1)$ signifying the relative timing of the regime change point and the bracket denotes the integer part.

To test the null hypothesis of no cointegration, the ADF test is calculated by the corresponding t-test for the slope of $\hat{u}_{t-1}$ in a regression of $\Delta\hat{u}_t$ on $\hat{u}_{t-1}, \Delta\hat{u}_{t-1}, \ldots, \Delta\hat{u}_{t-k}$, where $\hat{u}_t$ signifies the estimated error term from regression (2). The Z_α and Z_t test statistics are based on the calculation of the bias-corrected first-order serial correlation coefficient estimate $\hat{\rho}^*$, defined as

$$\hat{\rho}^* = \frac{\sum_{t=1}^{n-1} \left(\hat{u}_t \hat{u}_{t+1} - \sum_{j=1}^{B} w(j/B)\hat{\gamma}(j) \right)}{\sum_{t=1}^{n-1} \hat{u}_t^2}, \tag{3}$$

[1] For cointegration, higher orders of integration for y_t and x_t are also allowed as long as they are of the same order of integration, and the linear combination is integration of one less order than y_t and x_t. In this paper we limit are study to y_t and x_t being at most integrated of degree one.

[2] By a regime shift it is meant that there is a change in both the intercept and the slope parameters.

where $w(\cdot)$ is a function providing kernel weights meeting the standard conditions for spectral density estimators, B (itself a function of n) is the bandwidth number satisfying the conditions $B \to \infty$ and $B/n^5 = \mathrm{O}(1)$, and $\hat{\gamma}(j)$ is an autocovariance function. The autocovariance function is defined by

$$\hat{\gamma}(j) = \frac{1}{n} \sum_{t=j+1}^{T} (\hat{u}_{t-j} - \hat{\rho}\hat{u}_{t-j-1})(\hat{u}_t - \hat{\rho}\hat{u}_{t-1}), \tag{4}$$

where $\hat{\rho}$ is the OLS estimate of the effect (without intercept) of $\hat{u}_{t-1}$ on $\hat{u}_t$. The Z_α and Z_t test statistics are defined as

$$Z_\alpha = n(\hat{\rho}^* - 1), \tag{5}$$

and

$$Z_t = \frac{(\hat{\rho}^* - 1)}{\left(\hat{\gamma}(0) + 2\sum_{j=1}^{B} w(j/B)\hat{\gamma}(j)\right)/\sum_1^{n-1} \hat{u}_t^2}, \tag{6}$$

where $\hat{\gamma}(0)+2\sum_{j=1}^{B} w(j/B)\hat{\gamma}(j)$ is the long-run variance estimate of the residuals of a regression of $\hat{u}_t$ on $\hat{u}_{t-1}$.[3] These three test statistics have nonstandard distributions. It should be mentioned that the asymptotic distribution of the ADF test statistic is identical to the distribution of the Z_t statistic.

Our applicable test statistics are the smallest values of these three tests across all values for τ_1 and τ_2, with $\tau_1 \in T_1 = (0.15, 0.70)$ and $\tau_2 \in T_2 = (0.15+\tau_1, 0.85)$. The idea behind choosing the smallest value for each test statistic is that the smallest value represents the empirical evidence against the null hypothesis.[4] These test statistics are defined as

$$ADF^* = \inf_{(\tau_1,\tau_2)\in T} ADF(\tau_1, \tau_2), \tag{7}$$

$$Z_t^* = \inf_{(\tau_1,\tau_2)\in T} Z_t(\tau_1, \tau_2), \tag{8}$$

$$Z_\alpha^* = \inf_{(\tau_1,\tau_2)\in T} Z_\alpha(\tau_1, \tau_2), \tag{9}$$

where $T = (0.15n, 0.85n)$. The idea to truncate the data by 15% on each side follows the foot-steps of Gregory and Hansen (1996). Based on the same logic we also let the distance between the two regime shifts be at least 15%.

[3] The estimation for the long-run variance utilizes an automatic bandwidth estimator and a prewhitened quadratic spectral kernel with a first-order autoregression for the prewhitening. For details, Gregory and Hansen (1996) refer us to Andrews (1991) and Andrews and Monahan (1992).

[4] See also Gregory and Hansen (1996).

Table 1 Approximate asymptotic critical values for tests of cointegration with two regime shifts

The number of independent variables	Test statistic	1% Critical value	5% Critical value	10% Critical value
$m = 1$	ADF^*, Z_t^*	−6.503	−6.015	−5.653
	Z_α^*	−90.794	−76.003	−52.232
$m = 2$	ADF^*, Z_t^*	−6.928	−6.458	−6.224
	Z_α^*	−99.458	−83.644	−76.806
$m = 3$	ADF^*, Z_t^*	−7.833	−7.352	−7.118
	Z_α^*	−118.577	−104.860	−97.749
$m = 4$	ADF^*, Z_t^*	−8.353	−7.903	−7.705
	Z_α^*	−140.135	−123.870	−116.169

3 Asymptotic distributions

The distribution of these test statistics can be expressed as functions of Brownian motions. However, since they are not given in closed forms, we generate the asymptotic critical values by simulation methods in a similar way as Gregory and Hansen (1996).

The asymptotic critical values are generated through response surface analysis. We run 10,000 replications for each sample size of $n = 50, 100, 150, 200, 250$ and 300 to produce the critical values, denoted by $CV(n, p, m)$, p is the percent quantile (significance level) and m signifies the number of independent variables in the regression. Then the following regression is estimated:

$$CV(n, p, m)_n = \lambda_0 + \lambda_1 n^{-1} + e_n. \tag{10}$$

The asymptotic critical value is presented by the estimated intercept ($\hat{\lambda}_0$) in the above regression. Critical values are produced for 1, 5 and 10% significance levels ($p = 0.01, 0.05$, and 0.10), and $m = 1, 2, 3$, and 4. The results are illustrated in Table 1. The simulations are conducted by a programme code written in Gauss. The specification of the response surface Eq. (10) was very satisfactory since the R-squared was higher than 98% for the majority of cases.

4 Size and power properties

The following data generating process is utilized following Engle and Granger (1987) and also applied by Banerjee et al. (1986) and Gregory and Hansen (1996):

$$\begin{aligned} y_t &= 1 + 2x_t + \varepsilon_t, \quad \varepsilon_t = \rho\varepsilon_{t-1} + u_t, \qquad u_t \sim NID(0, 1) \\ y_t &= x_t + \upsilon_t, \quad \upsilon_t = \upsilon_{t-1} + \omega_t, \qquad \omega_t \sim NID(0, 1), \end{aligned}$$

where x_t is a scalar variable ($m = 1$). If $\rho < 1$ then the two variables y_t and x_t are cointegrated. The size and power properties at the 5% significance level are

Table 2 Size and power of tests of cointegration with two regime shifts for $n = 100$ and $m = 1$

Test statistic	$\rho = 1.0$	$\rho = 0.5$	$\rho = 0.0$
ADF^*	0.032	0.593	0.985
Z_t^*	0.069	0.839	1.000
Z_α^*	0.069	0.839	1.000

Notes: Size and power properties are investigated at the 5% significance level. The column corresponding to $\rho = 1.0$ represents size and the other two columns represent power

Table 3 The results of tests for cointegration between the US and the UK during 1989–1999

Test statistic	Estimated test value	1% Critical value	5% Critical value	10% Critical value
ADF^*	−16.264	−6.503	−6.015	−5.653
Z_t^*	−21.819	−6.503	−6.015	−5.653
Z_α^*	−488.173	−90.794	−76.003	−52.232

Notes: The critical values are collected from Table 2. Note that $m = 1$ in this application

presented in Table 2. The tests seem to have small size distortions and very good power properties.

5 An application

The test statistics introduced in this paper are applied to testing the integration between the US and UK financial markets. Weekly data is used during the period 1989–1999. The US S&P 500 index and the UK FTSE 100 index are used. The variables are transformed into their natural logarithmic form. The source of the data is the *Ecowin* data-base. Each variable was tested for a unit root. The results of the tests for cointegration using the tests presented by Eqs. 7–9 are presented in Table 3. It should be mentioned that before testing for cointegration we also tested each variable for a unit root. The results, not presented here, showed that each variable is integrated to the first order. As can be seen from the results of the test for cointegration presented in Table 3, the estimated test value is much higher than the critical value at the one percent significance level in absolute terms. Thus, the null hypothesis of no cointegration is strongly rejected. The first unknown break selected by our method was in the beginning of 1991, which corresponds to the first Gulf War. The collapse of the Soviet Union and the start of the transition period for the East European economies might also explain this first structural break that is found. The other structural break was found to be at the end of 1992. Our conjecture is that this break is related to the exchange rate crisis, which many European countries including the UK experienced during 1992.

We also estimated the parameters by running the regression presented by Eq. 2. The dependent variable is the log of UK FTSE 100 index (y_t) and the independent variable is the log of S&P 500 index (x_t). Since the variables are in logarithmic form the estimated slope represents an elasticity. The estimated values of the parameters are presented in Table 4. Each of the estimated parameter value is significant except for the

Table 4 The estimated values of the parameters

	α_0	α_1	α_2	β_0	β_1	β_2
Estimated parameter values	2.952	1.217	−9.280	0.817	−0.581	1.167
t values	2.052	0.844	−39.316	3.309	−2.351	40.611

first break in the intercept. The estimated elasticity of UK FTSE 100 index with regard to S&P 500 index is equal to 0.817. This elasticity decreased by 0.581during the first break period and it increased by 1.167 during the second period. Thus, the integration between the financial markets during the second break increased. The implication of this empirical finding is the potential portfolio diversification benefits have decreased between the two markets.

6 Conclusions

This paper focuses on the subject of testing for long-run equilibrium relationships (cointegration) between time series variables of interest when this potential relationship may shift twice during the period of study with unknown timing that is determined by the underlying data. Three residual-based tests are introduced for this purpose. Asymptotic critical values are produced for these test statistics via simulation methods. We also investigate the size and power properties of each test statistic via Monte Carlo experiments, which show that each test has a small size distortion and very good power properties. An application to the testing of financial market integration between the US and the UK markets is provided. The results show that the markets establish a long-run steady-state relation when two structural breaks are taken into account and the integration between them has increased during the last period. Our method identifies the two structural breaks as the following. The first break has occurred in the beginning of 1991, which might be explained by the first Gulf War. The other break has taken place at the end of 1992, which might be due to the exchange rate crisis that many European countries including the UK experienced during that period. The result has implications about the international portfolio diversification possibilities.

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Appendix

Asymptotic distributions

Under the null hypothesis of no cointegration with one unknown regime shift the Z_α^* and Z_t^* test statistics (presented by Eqs. 8, 9) have the following asymptotic distributions according to Gregory and Hansen (1996):

$$Z_t^* \to_d \inf_{\tau \in n} \int_0^1 W_\tau dW_\tau / \left[\int_0^1 W_\tau^2 \right]^{\frac{1}{2}} \left[1 + k' \varphi_\tau k_\tau \right]$$

And

$$Z_\alpha^* \to_d \inf_{\tau \in n} \int_0^1 W_\tau dW_\tau / \int_0^1 W_\tau^2$$

where

$$W_\tau(r) = W_1(r) - \int_0^1 W_1 dW_{2\tau}' \left[\int_0^1 W_2 dW_{2\tau}' \right]^{-1} W_{2\tau}(r),$$

$$k_\tau = \left[\int_0^1 W_2 dW_{2\tau}' \right]^{-1} \int_0^1 W_{2\tau} dW_1',$$

$$W_\tau(r) = \left[1, \varphi_\tau(r), W_2'(r), W_2'(r)\varphi_\tau(r) \right]',$$

$$D_\tau = \begin{pmatrix} 0 & 0 & 0 \\ 0 & I_m & (1-\tau)I_m \\ 0 & (1-\tau)I_m & (1-\tau)I_m \end{pmatrix}$$

where W is a vector Brownian motion for each particular case. For a proof see Gregory and Hansen (1996). The distributions of these tests when two unknown regime shifts prevail are similar to the above distributions except τ is a two dimensional vector as (τ_1, τ_2).

It should be pointed out that the distribution of ADF^* test statistic (presented by Eq. 7) is equivalent to the asymptotic distribution of Z_t^* statistic. This point was verified by the simulation results also.

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