

In the Testing for Cointegration section describing Hansen's version of the L_c :

Equation 25.24 reads $\hat{S}_t = \sum_{r=1}^t \hat{s}_t$ but the index on small "s" should be "r" (or visa versa, summing over t rather than r), and the terminal value should be upper-case T.

In Equation 25.26 you use the trace operator. But if I am following the details then in single equation estimation \hat{S}_t is a k by 1 vector (where "k" is the number of cointegrating equation regressors including constant and trends). And then $\hat{S}_t' G^{-1} \hat{S}_t$ is a scalar, so the trace operator is not needed. Also, the summation index is again denoted inconsistently as both "r" and "t".

Some more general comments:

Hansen (1992) allows y_t to be a vector, considering a system of cointegrating regressions. So in Hansen your Z_t is one observation of a matrix of values, which then forces use of summation notation over observations. But for this reader it would aid clarity to orient the discussion to single equation estimation. Then there are steps in FMOLS where it is possible to drop the summation notation and replace it with equivalents of $(X'X)^{-1}X'y$.

For instance, let k = the number of variables (including constant and trends) in the cointegrating regression. Then define the T-1 by k matrix of these variables as Z, i.e. Z without a time subscript equals the matrix of cointegrating equation regressors stacked in the usual manner, omitting your $Z'_{t=1}$, since there is no observation of y_t^+ for t=1. And define X as the usual matrix of regressors (including any constant and trends), so X includes the first observation omitted from matrix Z. And define y^+ as the T-1 vector of all observations of y_t^+ . And let N = T-1. Then in the documentation for Estimating a Cointegrating Regression, equation 25.8 can be written as

$$\begin{pmatrix} \hat{\beta} \\ \hat{\gamma}_1 \end{pmatrix} = (X'X)^{-1} \left(Z'y^+ - N \begin{bmatrix} \hat{\lambda}_{12}^+ \\ 0 \end{bmatrix} \right)$$

which shows how close FMOLS is to simple OLS algebra.

If you want to retain the summation notation and the use of Z_t then in Equation 25.8 the index on the second summation term actually starts with t=2, i.e.

$$(25.26) \begin{pmatrix} \hat{\beta} \\ \hat{\gamma}_1 \end{pmatrix} = \left(\sum_{t=1}^T Z_t Z_t' \right)^{-1} \left(\sum_{t=2}^T Z_t y_t^+ - (T-1) \begin{bmatrix} \hat{\lambda}_{12}^+ \\ 0 \end{bmatrix} \right)$$

Documentation for Hansen's Instability Test (L_c)

In Equation 25.26 the summation cannot begin at $r=1$ (nor $t=1$). Small s_t and large S_t are not defined for $t=1$. I found that not allowing for this detail in the exposition confusing, leading me to think I had missed something along the way. It would be helpful if the equation was written as summing over $t=2$ to T . If you want to allow for the possibility of more lags in the specification of the x -variable processes, then in the documentation it would be helpful to define some variable equal to the number of lags, and then start the summation at that value, i.e. something like $t = L$ to T , where L is the number of lags in the levels model of the regressors.

This leads me to the juncture at which I am less confident that I understand the procedure. In FMOLS I have been able to write a program reproducing the b -hats calculated by Eviews with a difference on the order of 10^{-15} , essentially zero. But in smaller samples (200 or less) my calculations of the L_c differ from those in Eviews by something on the order of 0.005. To avoid an even larger difference I did two things in my calculation of the L_c . First, in Equation 25.26 I used a scaling factor of $1/T$ rather than scaling by the number of terms in the sum (which is $T-1$). Second, as in FMOLS estimation I used a starting index in Equation 25.27 (for matrix G) different from that in Equation 25.26 (L_c). For 25.27 (G) the starting index is $t=1$, but for Equation 25.26 (L_c) the starting index is $t=2$ (implying a starting index $t=L$, if additional lags are allowed in specifying the x -var process). I have attached my program, in the hope that you may be willing to take a look at why I am not more accurately reproducing the L_c from Eviews. My comparison of values as recorded by hand (since in Eviews Hansen's L_C test is available only as an interactive command) is here:

DGP and model is $x_1, x_2, \text{constant}$

N	Lc_eviews	My calculation	difference	ratio
30	0.2638555	0.2615900	-0.002	0.991
30	0.1473780	0.1537722	0.006	1.043
30	0.2206666	0.2249788	0.004	1.020
30	0.3751272	0.3821771	0.007	1.019
30	0.8029390	0.8250586	0.022	1.028
60	0.1930272	0.1953072	0.002	1.012
60	0.4068926	0.4154204	0.009	1.021
60	0.1541090	0.1545781	0.000	1.003
100	0.3471111	0.3520841	0.005	1.014
100	0.4043402	0.3996603	-0.005	0.988
100	0.1512070	0.1527674	0.002	1.010
200	0.1935415	0.1917040	-0.002	0.991
200	0.4624064	0.4624083	0.000	1.000
400	0.2112259	0.2118362	0.001	1.003
400	0.6593625	0.6617915	0.002	1.004
1000	0.5302610	0.5300058	0.000	1.000
1000	0.1593516	0.1592906	0.000	1.000

Finally, if one wants to also calculate the FMOLS version of the breakpoint F, sup-F, mean-F etc then one will be using the M-matrix defined in Hansen. Using Hansen's formulation of the L_c (which employs $M_{TT} = M$), then in the case of a single cointegrating equation one can avoid summation notation. Letting S (without a time subscript) equal the k by $T-1$ matrix of observations of S_t , and letting $M = X'X$ (where again this X includes constant and trends) then the calculation is

$$L_c \text{ (scaled)} = (1/N) \text{tr}[M^{-1}(1/\hat{\omega}_{1,2})SS']$$

I am hoping that at some point Eviews will add the cointegrating equation version of the Sup- and Mean-F tests, in which case having introduced $M = \text{Hansen's } M_{TT}$ will make the documentation flow smoothly.

Hope some of this has been worth your time.