# MARCH-LM and MARCH-Portmanteau tests

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This document explains how to perform multivariate $ARCH$ tests using an Eviews add-in. The tests are designed to detect the existence of $ARCH$ effects and their structure, they can be applied before and after the estimation of a $MGARCH(p,q)$ model with any functional form. The add-in is based on [1].

## 1 MARCH tests

Both tests are designed to detect the existence of $ARCH$ effects in the residuals of a VAR or VEC model with $K$ endogenous variables before the $MGARCH(p,q)$ is fitted to the data, also the tests are designed to check for remaining $ARCH$ effects in the residuals after the estimation of the $MGARCH(p,q)$ model. The tests for remaining $ARCH$ effects are of particular importance, given that the structure of the $MGARCH(p,q)$ models are subject to the curse of dimensionality so that large order models involve large number of parameters making the estimation computationally difficult. Therefore to first fit a multivariate $GARCH$ model a low order of it is suggested to overcome these problems, nevertheless the conditionally heteroskedastic dynamics may not be well captured by this functional form and a higher order model is needed, both tests can light on the true of conditionally heteroskedastic dynamics.

### 1.1 MARCH-LM test

The $MARCH-LM$ test is based on the following system of auxiliary equations estimated from the residuals of a VAR or VEC model with $K$ endogenous variables, or from the standardized residuals[1] after the estimation of the $MGARCH(p,q)$ model with any functional form[2].

$$vech(u_t u_t^{'}) = \beta_0 + \beta_1 vech(u_t u_t^{'}) + \cdots + \beta_q vech(u_{t-q} u_{t-q}^{'}) + e_t$$

The system is estimated by $OLS$. Where vech denotes the half vectorization operator, $\beta_0$ is a vector of size $1 \times \frac{K(K+1)}{2}$ , and the matrices $\beta_i$ are of size $\frac{K(K+1)}{2} \times \frac{K(K+1)}{2}$ for$i = 1, ..., q$ . If the all matrices $\beta_i$ are equal to zero then there is not $ARCH$ in the residuals. Hence with the auxiliary system of equations the following hypothesis are tested.

$$H_0 : \beta_1 = \cdots = \beta_q = 0$$

The estimation of variance covariance matrix of the $e_t$ vector is denoted by $\Sigma_{vech,q}$ and for $q = 0$ the matrix is denoted by $\Sigma_0$ . The $ARCH-LM$ statistical is constructed as following.

$$LM_{MARCH(q)} = \frac{1}{2}TK(K+1) - Ttr(\Sigma_{vech,q}\Sigma_0^{-1}) \sim \chi^2(\frac{qK^2(K+1)^2}{4})$$

The $\chi^2$ degrees of freedom of the distribution of the statistic are the same as the number of parameters of the auxiliary equation excluding the constant vector. Therefore the test is not suitable for large $q$ unless the sample size is very large too.

[1]The add-in contains three options to standardize the residuals by the conditional variances and covariances: square root of covariance, square root of correlation and the Cholesky decomposition. These are way to calculate the matrix $\Sigma_t^{-1/2}$ to estimate the standardized residuals as $\varepsilon_t = \Sigma_t^{-1/2} u_t$.

[2]Eviews contains three functional forms of the $MGARCH$ models: diagonal VECH, Constant Conditional Correlation (CCC) and diagonal BEKK.

### 1.2 MARCH Portmanteau test

The second test included in the add-in is the portmanteau test, its main objective is to prove if the process $x_t = vech(u_t u_t^{'})$ has no serial correlation or in the same way no $ARCH$ effects. The test is based on the following two statistics, the second one it is their modified version which is in spirit the multivariate version of the Ljung-Box Q statistic.

$$Q_h^{ARCH} = T\sum_{i=1}^{h} tr(C_i^{'}C_0^{-1}C_iC_0^{-1})$$

$$Q_h^{ARCH*} = T^2\sum_{i=1}^{h}(T-i)^{-1}tr(C_i^{'}C_0^{-1}C_iC_0^{-1})$$

Where $C_i$ are the covariance matrices estimated as $C_i = T^{-1}\sum_{t=i+1}^{T}(x_t-\overline{x})(x_{t-i}-\overline{x})'$ for$i = 0,1,...,h$, it is important to subtract the mean given that the $vech(u_t u_t^{'})$ does not have zero mean. The statistical follow an asymptotic $\chi^2(\frac{K^2(K+1)^2}{4}(h-P))$ distribution. Where $P$ is the lag order of the VAR(P) model fitted to the conditional mean.

# 2 The add-in

The add-in could be used via point and clicking from the global menu or via command line, the command are showed in following table.

*mgarch_tests(options)*

| Option | Command |
|---|---|
| *Name of the VAR o SYSTEM estimates* | *Object= Name of the object in the current workfile* |
| *Maximum lags to perform the tests (q and h)* | *Maxlags= Integer (default 1)* |
| *Standardize residuals using square root of covariance* | *cov* |
| *Standardize residuals using square root of correlation* | *cor* |
| *Standardize residuals using Cholesky* | *chol* |
| *Save VECH residuals* | *vech_residuals* |

Table 1: Command line

In the case that the user specify a $VAR$ object with estimates of a $VAR, BVAR$ or $VEC$ model both tests will be performed on the vector of residuals of this estimates. On the other hand if the user specify a $SYS$ object this must contain $MGARCH$ estimates of any kind and the test will be performed on the standardized residuals to check for remaining $ARCH$ effects. If the user want to save the $VECH$ residuals the a table and a graph of those will be showed.

# 3 Example using the add-in

For this example a $VAR(2)-VECH(1,1)$ was simulated for $T = 25.000$ with EViews as:

$$\begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.2 \end{bmatrix}\begin{bmatrix} x_{1t-1} \\ x_{2t-1} \end{bmatrix} + \begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.2 \end{bmatrix}\begin{bmatrix} x_{1t-2} \\ x_{2t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$u_t = \Sigma_t^{1/2}\varepsilon_t$$

$$\Sigma_t^{1/2} = P\Lambda^{1/2}P'$$

$$Vech(\Sigma_t) = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.5 \end{bmatrix} + \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}\begin{bmatrix} u_{1t-1}^2 \\ u_{1t-1}u_{2t-1} \\ u_{2t-1}^2 \end{bmatrix} + \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}\begin{bmatrix} \sigma_{1t-1}^2 \\ \sigma_{1t-1}\sigma_{2t-1} \\ \sigma_{2t-1}^2 \end{bmatrix}$$

ARCH-LM and ARCH-Portmanteau test for MGARCH models
Null Hypothesis: no M-ARCH effects up to lag q

| Lag | LM Stat | P-value | Df | Portmanteau Stat | P-value* | Adj Portmanteau Stat | P-value* | Df* |
|---|---|---|---|---|---|---|---|---|
| 1 | 3121.5764 | 0.00 | 9 | 2933.4561 | NA | 2933.5734 | NA | NA |
| 2 | 3290.7177 | 0.00 | 18 | 3465.8026 | NA | 3465.9625 | NA | NA |
| 3 | 3304.8885 | 0.00 | 27 | 3569.3578 | 0.00 | 3569.5302 | 0.00 | 9 |
| 4 | 3308.1320 | 0.00 | 36 | 3589.0675 | 0.00 | 3589.2430 | 0.00 | 18 |
| 5 | 3313.0074 | 0.00 | 45 | 3599.7149 | 0.00 | 3599.8926 | 0.00 | 27 |
| 6 | 3321.2296 | 0.00 | 54 | 3606.5084 | 0.00 | 3606.6877 | 0.00 | 36 |
| 7 | 3325.8166 | 0.00 | 63 | 3614.9818 | 0.00 | 3615.1635 | 0.00 | 45 |
| 8 | 3335.3088 | 0.00 | 72 | 3625.7579 | 0.00 | 3625.9430 | 0.00 | 54 |
| 9 | 3338.3001 | 0.00 | 81 | 3628.6520 | 0.00 | 3628.8381 | 0.00 | 63 |
| 10 | 3343.2827 | 0.00 | 90 | 3637.5483 | 0.00 | 3637.7380 | 0.00 | 72 |
| 11 | 3352.0598 | 0.00 | 99 | 3646.8475 | 0.00 | 3647.0413 | 0.00 | 81 |
| 12 | 3357.1406 | 0.00 | 108 | 3657.4680 | 0.00 | 3657.6669 | 0.00 | 90 |

*The probability of the test can only be calculated for lags greater than the VAR lag order

Figure 1: MARCH tests on the VAR(2) residuals

ARCH-LM and ARCH-Portmanteau test for MGARCH models
Null Hypothesis: no M-ARCH effects up to lag q

| Lag | LM Stat | P-value | Df | Portmanteau Stat | P-value* | Adj Portmanteau Stat | P-value* | Df* |
|---|---|---|---|---|---|---|---|---|
| 1 | 12.4451 | 0.19 | 9 | 1.6928 | NA | 1.6928 | NA | NA |
| 2 | 14.6519 | 0.69 | 18 | 6.2451 | NA | 6.2456 | NA | NA |
| 3 | 16.8814 | 0.93 | 27 | 10.8656 | 0.29 | 10.8666 | 0.28 | 9 |
| 4 | 20.2475 | 0.98 | 36 | 16.2116 | 0.58 | 16.2134 | 0.58 | 18 |
| 5 | 21.3774 | 1.00 | 45 | 18.8990 | 0.87 | 18.9013 | 0.87 | 27 |
| 6 | 28.6624 | 1.00 | 54 | 27.3471 | 0.85 | 27.3515 | 0.85 | 36 |
| 7 | 37.0186 | 1.00 | 63 | 38.1725 | 0.75 | 38.1800 | 0.75 | 45 |
| 8 | 43.9291 | 1.00 | 72 | 47.3041 | 0.73 | 47.3144 | 0.73 | 54 |
| 9 | 46.8612 | 1.00 | 81 | 52.5388 | 0.82 | 52.5510 | 0.82 | 63 |
| 10 | 54.1054 | 1.00 | 90 | 62.1500 | 0.79 | 62.1660 | 0.79 | 72 |
| 11 | 60.9675 | 1.00 | 99 | 71.2681 | 0.77 | 71.2882 | 0.77 | 81 |
| 12 | 73.2948 | 1.00 | 108 | 86.0297 | 0.60 | 86.0569 | 0.60 | 90 |

*The probability of the test can only be calculated for lags greater than the VAR lag order
Standarized residuals using square root of covariance

Figure 2: MARCH tests of the VAR(2)-VECH(1,1) standardized residuals

Where $\varepsilon_t \sim N(0, I_2)$, $P$ is a $KxK$ matrix with the eigenvectors of $\Sigma_t$ and $\Lambda^{1/2}$ is a $KxK$ diagonal matrix with the square root of the eigenvalues of $\Sigma_t$ in its elements. As can be seen the simulated time series $x_{1t}$ and $x_{2t}$ follow a stable $VAR(2)$ with diagonal $VECH(1,1)$ residuals. In figure 1 the results of the $MARCH$ tests applied to the estimated residuals of the $VAR(2)$ are showed. As can be seen the null of no $MARCH$ effects is rejected, therefore an $MGARCH$ model is suggested. The simulated data is available in the examples workfile and the results can be obtain with the command *mgarch_tests(object=var2,maxlags=12)*.

In figure 2 the tests results for the standardized residuals of the estimated $VECH(1,1)$ are showed. As can be seen the null of no $MARCH$ remaining effects is nor rejected for any lag and the estimated model captures the conditional heterosedasticity of the time series. These results can be obtained with the command *mgarch_tests(object=mgarch,maxlags=12)* where *mgarch* contains the estimates of the full model.

# References

[1] Helmut Lütkepohl. *New introduction to multiple time series analysis*. Springer Science & Business Media, 2005.