

# Seasonal Unit Root Test

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This document explains how to perform a seasonal unit root test using an Eviews add-in. The add-in test works with biannual, quarterly and monthly data. The add-in contains the option to obtain the p-value via Monte Carlo methods.

## 1 HEGY Test

The seasonal unit root test of the add-in is based on [4], which introduces the test but concentrates in quarterly time series. The test for biannual and monthly data can be found in [2, 3].

The goal of the test is to determine what class of seasonal processes is the generator of the seasonality, these could be: a deterministic seasonal processes, a stationary seasonal processes or an integrated seasonal processes. A series  $s_t$  is an integrated seasonal processes if it has a seasonal unit root and a peak at any seasonal frequency in its spectrum (excluding the zero frequency), more generally it is integrated of order  $d$  at frequency  $\theta$  i.e.  $s_t \sim I_\theta(d)$ .

The method of performing the test is explained below for biannual, quarterly and monthly time series.

### 1.1 Biannual

For these temporal aggregation frequency the test consist of estimating the parameters in equation (1) by OLS.

$$\Phi(L)s_{3,t} = \mu_t + \pi_1 s_{1,t-1} + \pi_2 s_{2,t-1} + e_t \quad (1)$$

Where  $\mu_t$  are deterministic regressors, in the add-in four options of these regressors are included: intercept, intercept and trend, intercept and seasonal dummies and intercept, trend and seasonal dummies, whenever the seasonal dummies are included these will be centered dummy variables, and where:

$$s_{3,t} = (1 - L^2)s_t \quad (2)$$

$$s_{1,t} = (1 + L)s_t \quad (3)$$

$$s_{2,t} = -(1 - L)s_t \quad (4)$$

And  $\Phi(L)$  is a polynomial in the lag operator, in the add-in the order of  $\Phi(L)$  is selected using information criteria. If  $\pi_1 = 0$  then the presence of the non-seasonal unit root cannot be rejected. If  $\pi_2 = 0$  then the presence of a seasonal unit root cannot be rejected. This hypothesis can be tested using (one-sided) t-test, nevertheless the distribution of these tests are nonstandard.

### 1.2 Quarterly

The test consist of estimating the parameters in equation (5) by OLS.

$$\Phi(L)s_{4,t} = \mu_t + \pi_1 s_{1,t-1} + \pi_2 s_{2,t-1} + \pi_3 s_{3,t-1} + \pi_4 s_{3,t-2} + e_t \quad (5)$$

Where:

$$s_{4,t} = (1 - L^4)s_t \quad (6)$$

$$s_{1,t} = (1 + L + L^2 + L^3)s_t \quad (7)$$

$$s_{2,t} = -(1-L)(1+L^2)s_t \quad (8)$$

$$s_{3,t} = -(1-L^2)s_t \quad (9)$$

For this temporal aggregation there could be two seasonal unit roots. The hypothesis on the coefficients  $\pi_1$  and  $\pi_2$  are tested using t-tests as before. If  $\pi_3 = \pi_4 = 0$  then the presence of a seasonal unit root cannot be rejected, these hypothesis can be tested using a joint F-test, which has a nonstandard distribution.

### 1.3 Monthly

For this the test consist of estimating equation (10) by OLS.

$$\Phi(L)s_{8,t} = \mu_t + \pi_1 s_{1,t-1} + \pi_2 s_{2,t-1} + \pi_3 s_{3,t-1} + \pi_4 s_{3,t-2} + \pi_5 s_{4,t-1} + \pi_6 s_{4,t-2} + \pi_7 s_{5,t-1} + \quad (10)$$

$$\pi_8 s_{5,t-2} + \pi_9 s_{6,t-1} + \pi_{10} s_{6,t-2} + \pi_{11} s_{7,t-1} + \pi_{12} s_{7,t-2} + e_t$$

Where.

$$s_{8,t} = (1-L^{12})s_t \quad (11)$$

$$s_{1,t} = (1+L)(1+L^2)(1+L^4+L^8)s_t \quad (12)$$

$$s_{2,t} = -(1-L)(1+L^2)(1+L^4+L^8)s_t \quad (13)$$

$$s_{3,t} = -(1-L^2)(1+L^4+L^8)s_t \quad (14)$$

$$s_{4,t} = -(1-L^4)(1-\sqrt{3}L+L^2)(1+L^2+L^4)s_t \quad (15)$$

$$s_{5,t} = -(1-L^4)(1+\sqrt{3}L+L^2)(1+L^2+L^4)s_t \quad (16)$$

$$s_{6,t} = -(1-L^4)(1-L^2+L^4)(1-L+L^2)s_t \quad (17)$$

$$s_{7,t} = -(1-L^4)(1-L^2+L^4)(1+L+L^2)s_t \quad (18)$$

The hypothesis on the coefficients  $\pi_1, \pi_2, \pi_3$  and  $\pi_4$  are tested as before. If  $\pi_i = \pi_{i+1} = 0$  then the presence of a seasonal unit root cannot be rejected for  $i = 5, 7, 9, 11$ , this hypothesis can be tested using a joint F-test, which has a nonstandard distribution.

## 2 Critical values and Monte Carlo

The add-in contains two options to obtain a rule to decide if the null must be rejected or not. The first consist to compare the value of the statistics with the critical values tabulated in [3, 4], which are showed in the add-in. And the second consist on simulate the distributions of the statistics using the Monte Carlo method, the steps for the simulations performed in the add-in are showed below.

1. Simulate the process  $s_t = s_{t-s} + e_t$  where  $e_t \sim nid(0, 1)$ ,  $s = 2$  for the biannual test,  $s = 4$  for the quarterly test and  $s = 12$  for the monthly test. The simulated process contain the same number of observations than the tested time series, this gives more precision than compare the statistics with the tabulated critical values, given that they are available only for some values of the number of observations.
2. Obtain the value of the statistics in accordance with the temporal aggregation of the data.
3. Repeat steps 1 and 2 a one preferably large number of times.
4. Obtain the associated probability of the statistical considering their empirical distribution.

### 3 Using the add-in

The add-in can be used via point and clicking or via command line. Some examples will be showed using data of the money aggregate M1 of Colombia and of the critical radio frequencies (CRF) in Washington D.C. both present a strong seasonal pattern, the time series are plotted in figure 1. A workfile with the data comes with the installation.

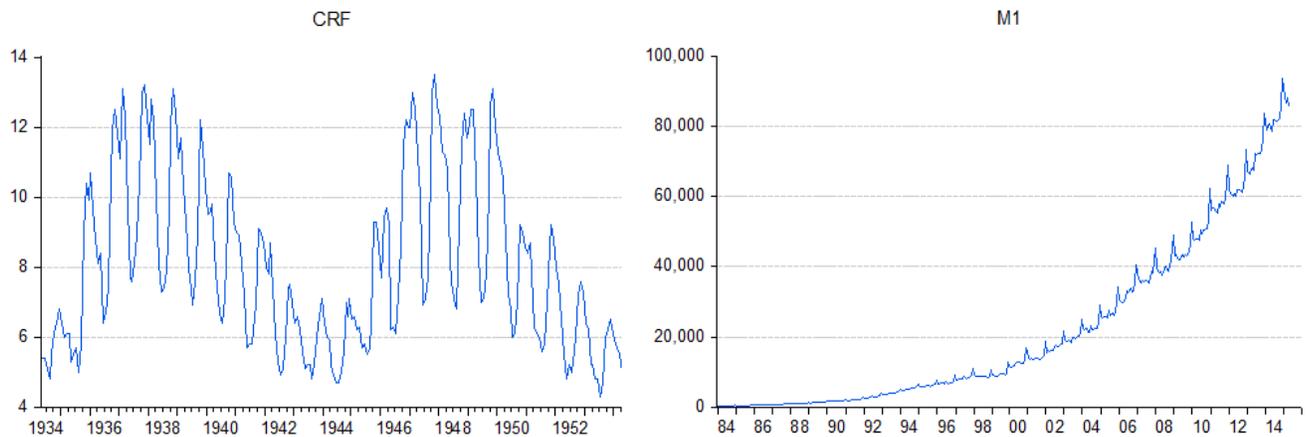


Figure 1: CRF and M1

After installing the add-in the user has to click in the time series object `proc->Add-ins->HEGY` to perform the test. Then the window showed in figure 2 will appear. As mentioned the add-in contains several options of deterministic regressors.

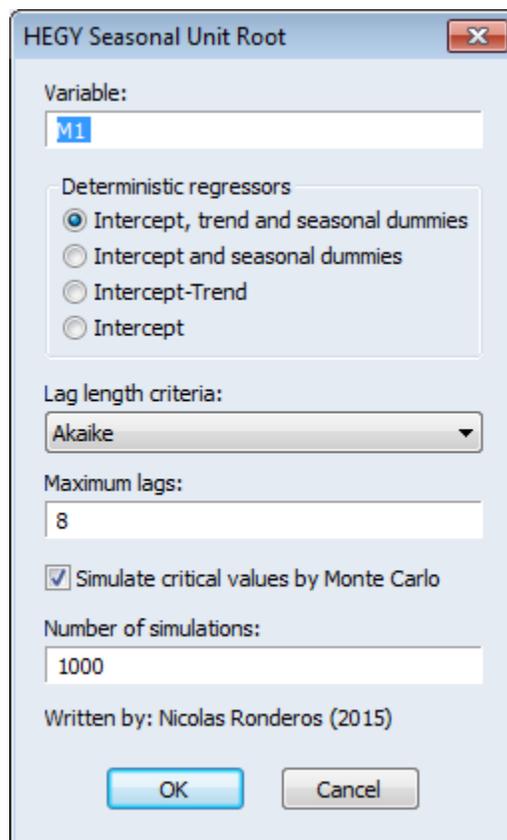


Figure 2: Add-in window

To access the add-in via command line the user just has to follow the instructions given in table 1. The defaults of the command line are the same as the ones showed in figure 2.

<i>series_name.hegy(options)</i>	Options
<i>Intercept,trend and seasonal dummies</i>	<i>itd</i>
<i>Intercept and seasonal dummies</i>	<i>id</i>
<i>Intercept and trend</i>	<i>it</i>
<i>Intercept</i>	<i>i</i>
<i>Akaike criteria to select optimum lag length</i>	<i>aic</i>
<i>Schwarz criteria to select optimum lag length</i>	<i>schwarz</i>
<i>Hannan-Quinn criteria to select optimum lag length</i>	<i>hq</i>
<i>Maximum lags to test</i>	<i>maxlag=8</i>
<i>Simulate via Monte Carlo</i>	<i>mc</i>
<i>Number of simulations</i>	<i>simulations=1000</i>

Table 1: Command line

Seasonal Unit Root test   HEGY		
Null	Simulated P-value*	Statistical
Nonseasonal unit root (Zero frequency)	1.000000	2.116130
Seasonal unit root (2 months per cycle)	0.670106	-1.198009
Seasonal unit root (4 months per cycle)	0.798418	1.126974
Seasonal unit root (2.4 months per cycle)	0.057313	5.890169
Seasonal unit root (12 months per cycle)	0.216937	3.952111
Seasonal unit root (3 months per cycle)	0.674464	1.740455
Seasonal unit root (6 months per cycle)	0.896129	0.687784

\*Monte Carlo Simulations: 2000  
Selected lag using hq criteria: 4

Figure 3: HEGY test on M1

Seasonal Unit Root test   HEGY		
Null	Critical values 5%*	Statistical
Nonseasonal unit root (Zero frequency)	-2.760000	-2.027506
Seasonal unit root (2 months per cycle)	-2.760000	-4.458383
Seasonal unit root (4 months per cycle)	6.270000	15.34461
Seasonal unit root (2.4 months per cycle)	6.280000	22.76195
Seasonal unit root (12 months per cycle)	6.210000	8.165683
Seasonal unit root (3 months per cycle)	6.220000	18.71443
Seasonal unit root (6 months per cycle)	6.210000	7.225980

\*Franses and Hobijn (1997)  
Selected lag using aic criteria: 2

Figure 4: HEGY test on CRF

For example, to perform the unit root test on M1 via command line the user has to use the following command *m1.hegy(hq,maxlag=6,mc,simulations=2000)*. The results of the test are showed in figure 3, it can be seen that

there is evidence of seasonal unit roots at almost all seasonal frequencies, therefore to model the nonstationary seasonality of this series a filter of the form  $(1 - L^{12})M1_t$  is suggested.

The non-seasonal unit root statistic corresponds to the t-statistic of  $\pi_1$ , the seasonal unit root statistic with 2 month per cycle corresponds to  $\pi_2$ , the statistics reported by the add-in corresponds respectably to the tested coefficient or pair of coefficients, this also holds for the other temporal aggregations.

The HEGY test on CRF is showed in figure 4, this was carried out with the following command `crf.hegy(id,aic,maxlag=4)`. It can be seen that there is evidence of one unit root at zero frequency, therefore to model the time series the usual first difference filter is suggested.

## References

- [1] J Joseph Beaulieu and Jeffrey A Miron. Seasonal unit roots in aggregate us data. *Journal of econometrics*, 55(1):305–328, 1993.
- [2] Philip Hans Franses. Seasonality, non-stationarity and the forecasting of monthly time series. *International Journal of Forecasting*, 7(2):199–208, 1991.
- [3] Philip Hans Franses and Bart Hobijn. Critical values for unit root tests in seasonal time series. *Journal of Applied Statistics*, 24(1):25–48, 1997.
- [4] Svend Hylleberg, Robert F Engle, Clive WJ Granger, and Byung Sam Yoo. Seasonal integration and cointegration. *Journal of econometrics*, 44(1):215–238, 1990.